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Similarity and Distance Measures of Picture Fuzzy Sets with Application in Pattern Recognition

Research Article

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ABSTRACT

Picture fuzzy set is the latest influential conception to conduct ambiguous data effectively. The distance and similarity measures are two dominant concepts to calculate the characteristics between two PFSs. The similarity measure evaluates the closeness between the two PFSs where the larger similarity measure corresponds to the closer degree of two PFSs. In this article, a new similarity measure is developed considering the influence of the abstain and refusal groups as each of these influences the results. Finally, a pattern recognition application is illustrated to justify the effectiveness of this new technique and compare the results with other existing methods.

Keywords: *Picture fuzzy sets; Similarity measure; Distance measure; Pattern recognition*

1 Introduction

In the present world, many situations are to deal where the data are more imprecise than precise. To deal with these uncertain situations Zadeh (Zadeh1965) introduced the notion of fuzzy set theory in 1965. The fuzzy set is the generalization of classical set theory and takes into account membership degree of an element and the non-membership degree is the direct complement of the membership degree. However, many applications in real life found some problems because of considering the value of non- membership degree. To over come this problem, Atanassov (Atanassov 1986) developed the concept of intuitionistic fuzzy set by allowing for the membership and non-membership degrees of an element in 1986, where the non-membership degree is not the direct complement of the membership degree. But, the degree of hesitation of an element in an intuitionistic fuzzy set revealed a dilemma. In 2013, Cuong and Kreinovich (Cuong et al. 2013, Cuong 2014) introduced the notion of picture fuzzy set

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which is a powerful tool to deal with vagueness and uncertainty including the idea of positive, negative, and neutral membership degrees of an element. Many research works have been done since it's development. In 2013, Cuong (Cuong et al. 2013, Cuong 2014) gave the Hamming distance and the Euclidean distance between picture fuzzy sets. In 2016, Son (Son 2016) proposed a generalized distance measure between picture fuzzy sets and applied it to fuzzy clustering. In 2017, Dutta (Dutta 2017) discussed distance measures on picture fuzzy sets and applied in Medical diagnosis. Also Son (Son 2016) discussed picture distance measures to picture association measures for measuring analogousness in PFSs. Also many applications on distance and similarity measures have been done in resect years (see: Chau 2020, Cuong et al. 2013, Kadian 2021, Khan et al. 2021, Luo et al. 2020, Liu et al. 2019, Peng et al. 2017, Singh et al. 2018, Thao 2019, Wei 2017, Wei et al. 2018, Wei 2016). However, many measures of similarity between PFSs have been proposed earlier but did not consider the abstention and refusal groups influence. On the other side, some existing similarity measures provide the count-intuitive results in some situations or cannot identify which two picture fuzzy sets are closer. In this work, a new similarity measure based on the influence of the abstain and refusal groups is proposed. An application of pattern recognition is discussed and the results are compared with the results of existing methods.

2 Preliminaries

In this section, we recall some basic definitions for picture fuzzy sets which are used in later sections.

Definition 2.1: (Zadeh1965) A ***fuzzy set A*** in $X \neq \phi$ is defined by $A = \{(x, \mu_A(x)): x \in X\}$, where $\mu_A: X \to [0, 1]$.

Definition 2.2: (Atanassov 1986) An ***intuitionistic fuzzy set A*** in $X \neq \phi$ is defined by

$A = \{(x, \mu_A(x), \nu_A(x)): x \in X\}$, where $\mu_A: X \to [0, 1]$ and $\nu_A: X \to [0, 1]$.

The values $\mu_A(x)$ and $\nu_A(x)$ represent the membership and non-membership degrees of the element x to the set A respectively and $0 \leq \mu_A(x) + \nu_A(x) \leq 1; \forall x \in X$ and $\pi_A(x)$ is called the hesitancy degree and $\pi_A(x) = 1 - \left(\mu_A(x) + \nu_A(x)\right)$.

Definition 2.3: (Cuong et al. 2013, Cuong 2014) A ***picture fuzzy set A*** in $X \neq \phi$ is defined by

$$A = \left\{\left(x, \mu_A(x), \eta_A(x), \nu_A(x)\right): x \in X\right\},$$

where $\mu_A(x), \eta_A(x), \nu_A(x) \in [0, 1]$ are the degrees of positive, neutral and negative memberships of x in A and $0 \leq \mu_A(x) + \eta_A(x) + \nu_A(x) \leq 1; \forall x \in X$ and the refusal membership of x is $\pi_A(x) = 1 - \left(\mu_A(x) + \eta_A(x) + \nu_A(x)\right); \forall x \in X$.

Definition 2.4: (Singh et al. 2018) Let $s: PFS(X) \times PFS(X) \longrightarrow [0, 1]$ so that for any PFSs A, B and C of X, it satisfies the following four properties :

i. $0 \leq s(A, B) \leq 1$,
ii. $s(A, B) = 1$ iff $A = B$,
iii. $s(A, B) = s(B, A)$,
iv. $if\ A \subset B \subset C$, then $s(A, C) \leq s(A, B)$ and $s(A, C) \leq s(B, C)$.

Then s is called a similarity function of PFSs and $s(A, B)$ is called the similarity degree between the PFSs A and B.

Definition 2.5: (Son 2016) Let $d: PFS(X) \times PFS(X) \longrightarrow [0, 1]$ so that for any PFSs A, B and C of X, it satisfies the following four properties :

i. $0 \leq d(A, B) \leq 1$,
ii. $d(A, B) = 0$ iff $A = B$,
iii. $d(A, B) = d(B, A)$,
iv. $d(A, B) \leq d(A, C) + d(C, B)$.

Then, $d(A, B)$ is called the normalized distance between the PFSs A and B.

3. Similarity and Distance between two Picture Fuzzy Sets

Definition 3.1: Let $X = \{x_1, x_2, \cdots\cdots\cdots, x_n\}$ be a finite universal set and A and B are two

PFSs on X, the Minkowski similarity degree between A and B can be is

$$s_q(A,B) = 1 - \sqrt[q]{\frac{1}{2n}\sum_{j=1}^{n}\left[\left(\mu_A(x_j) - \mu_B(x_j)\right)^q + \left(\eta_A(x_j) - \eta_B(x_j)\right)^q + \left(\upsilon_A(x_j) - \upsilon_B(x_j)\right)^q + \left(\pi_A(x_j) - \pi_B(x_j)\right)^q\right]} \quad (1.1)$$

,where $\pi_A(x_j) = 1 - \mu_A(x_j) - \eta_A(x_j) - \upsilon_A(x_j)$ and $\pi_B(x_j) = 1 - \mu_B(x_j) - \eta_B(x_j) - \upsilon_B(x_j)$ $(j = 1, 2, \cdots\cdots\cdots, n)$; $q > 0$ indicates the distance parameter.

If $q = 1$, then it becomes the Hamming similarity degree A and B which is as follows:

$$s_1(A,B) = 1 - \frac{1}{2n}\sum_{j=1}^{n}\left[\left|\mu_A(x_j) - \mu_B(x_j)\right| + \left|\eta_A(x_j) - \eta_B(x_j)\right| + \left|\upsilon_A(x_j) - \upsilon_B(x_j)\right| + \left|\pi_A(x_j) - \pi_B(x_j)\right|\right] \quad (1.2)$$

If $q = 2$, then it becomes the Euclidean similarity degree A and B which is

$$s_2(A,B) = 1 - \sqrt{\frac{1}{2n}\sum_{j=1}^{n}\left[\left(\mu_A(x_j) - \mu_B(x_j)\right)^2 + \left(\eta_A(x_j) - \eta_B(x_j)\right)^2 + \left(\upsilon_A(x_j) - \upsilon_B(x_j)\right)^2 + \left(\pi_A(x_j) - \pi_B(x_j)\right)^2\right]} \quad (1.3)$$

If $q = +\infty$, then it becomes the Chebyshev similarity degree between A and B which is

$$s_{+\infty}(A,B) = 1 - \max_{1 \le j \le n}\left\{\frac{\left|\mu_A(x_j) - \mu_B(x_j)\right| + \left|\eta_A(x_j) - \eta_B(x_j)\right| + \left|\upsilon_A(x_j) - \upsilon_B(x_j)\right| + \left|\pi_A(x_j) - \pi_B(x_j)\right|}{2n}\right\} \quad (1.4)$$

The weights of each element may be considered for their importance. Let the weight of each element x_j $(j = 1, 2, \cdots\cdots\cdots, n)$ is ω_j and $\omega_j \in [0,1]$ and $\sum_{j=1}^{n}\omega_j = 1$.

The weighted Minkowski similarity degree between A and B is

$$\bar{s}_q(A,B) = 1 - \sqrt[q]{\frac{1}{2n}\sum_{j=1}^{n}\omega_j\left[\left(\mu_A(x_j) - \mu_B(x_j)\right)^q + \left(\eta_A(x_j) - \eta_B(x_j)\right)^q + \left(\upsilon_A(x_j) - \upsilon_B(x_j)\right)^q + \left(\pi_A(x_j) - \pi_B(x_j)\right)^q\right]} \quad (1.1w)$$

The weighted Hamming similarity degree A and B is

$$\bar{s}_1(A,B) = 1 - \frac{1}{2n}\sum_{j=1}^{n}\omega_j\left[\left|\mu_A(x_j) - \mu_B(x_j)\right| + \left|\eta_A(x_j) - \eta_B(x_j)\right| + \left|\upsilon_A(x_j) - \upsilon_B(x_j)\right| + \left|\pi_A(x_j) - \pi_B(x_j)\right|\right] \quad (1.2w)$$

The weighted Euclidean similarity degree between A and B is

$$\bar{s}_2(A,B) = 1 - \sqrt{\frac{1}{2n}\sum_{j=1}^{n}\omega_j\left[\left(\mu_A(x_j) - \mu_B(x_j)\right)^2 + \left(\eta_A(x_j) - \eta_B(x_j)\right)^2 + \left(\upsilon_A(x_j) - \upsilon_B(x_j)\right)^2 + \left(\pi_A(x_j) - \pi_B(x_j)\right)^2\right]} \quad (1.3w)$$

The weighted Chebyshev similarity degree between A and B is

$$\bar{s}_{+\infty}(A,B) = 1 - \max_{1 \le j \le n}\left\{\frac{\omega_j\left[\left|\mu_A(x_j) - \mu_B(x_j)\right| + \left|\eta_A(x_j) - \eta_B(x_j)\right| + \left|\upsilon_A(x_j) - \upsilon_B(x_j)\right| + \left|\pi_A(x_j) - \pi_B(x_j)\right|\right]}{2n}\right\} \quad (1.4w)$$

Obviously, if all weights $\omega_j = \frac{1}{n}$ $(j = 1, 2, \cdots\cdots\cdots, n)$ i.e., weights of all elements x_j are identical, then equations (1.1w) – (1.4w) are reduced to equations (1.1) – (1.4), respectively.

Example 1: Let $A = \{(x_1, 0.5,0.2,0.3), (x_2, 0.2,0.3,0.4), (x_3, 0.6,0.3,0.1)\}$ and

$B = \{(x_1, 0.6,0.3,01), (x_2, 0.5,0.2,0.2), (x_3, 0.4,0.4,0.1)\}$ be two PFSs on the universal set $X = \{x_1, x_2, x_3\}$. Assume that weights of the elements x_1, x_2 and x_3 are given as follows: $\omega_1 = 0.5, \omega_2 = 0.2$ and $\omega_3 = 0.3$.

From the equation (1.2), we get

$$s_1(A,B) = 1 - \frac{1}{2n}\sum_{j=1}^{n}\left[\left|\mu_A(x_j) - \mu_B(x_j)\right| + \left|\eta_A(x_j) - \eta_B(x_j)\right| + \left|\upsilon_A(x_j) - \upsilon_B(x_j)\right| + \left|\pi_A(x_j) - \pi_B(x_j)\right|\right]$$

$$= 1 - \frac{1}{2\times 3}\,[(|\mu_A(x_1) - \mu_B(x_1)| + |\eta_A(x_1) - \eta_B(x_1)| + |\upsilon_A(x_1) - \upsilon_B(x_1)| + |\pi_A(x_1) - \pi_B(x_1)|) + (|\mu_A(x_2) - \mu_B(x_2)| + |\eta_A(x_2) - \eta_B(x_2)| + |\upsilon_A(x_2) - \upsilon_B(x_2)| + |\pi_A(x_2) - \pi_B(x_2)|) + (|\mu_A(x_3) - \mu_B(x_3)| + |\eta_A(x_3) - \eta_B(x_3)| + |\upsilon_A(x_3) - \upsilon_B(x_3)| + |\pi_A(x_3) - \pi_B(x_3)|)]$$

$$= 1 - \frac{1}{6}\,[(|0.5 - 0.6| + |0.2 - 0.3| + |0.3 - 0.1| + |0.0 - 0.0|) + (|0.2 - 0.5| + |0.3 - 0.2| + |0.4 - 0.2| + |0.1 - 0.1|) + (|0.6 - 0.4| + |0.3 - 0.4| + |0.1 - 0.1| + |0.0 - 0.1|)]$$

$$= 1 - \frac{1}{6}\,[(0.1 + 0.1 + 0.2 + 0.0) + (0.3 + 0.1 + 0.2 + 0.0) + (0.2 + 0.1 + 0.0 + 0.1)]$$

$$= 1 - \frac{1}{6}\,[(0.1 + 0.1 + 0.2 + 0.0) + (0.3 + 0.1 + 0.2 + 0.0) + (0.2 + 0.1 + 0.0 + 0.1)]$$

$$= 1 - \frac{1}{6}\,[0.4 + 0.6 + 0.4] = 1 - \frac{1.4}{6} = 1 - 0.23 = 0.77$$

From the equation (1.3), we get

$$s_2(A,B) = 1 - \sqrt{\frac{1}{2n}\sum_{j=1}^{n}\left[\left(\mu_A(x_j) - \mu_B(x_j)\right)^2 + \left(\eta_A(x_j) - \eta_B(x_j)\right)^2 + \left(\upsilon_A(x_j) - \upsilon_B(x_j)\right)^2 + \left(\pi_A(x_j) - \pi_B(x_j)\right)^2\right]}$$

$$= 1 - \left\{\frac{1}{2\times 3}\left[\left\{\left(\mu_A(x_1) - \mu_B(x_1)\right)^2 + \left(\eta_A(x_1) - \eta_B(x_1)\right)^2 + \left(\upsilon_A(x_1) - \upsilon_B(x_1)\right)^2 + \left(\pi_A(x_1) - \pi_B(x_1)\right)^2\right\} + \left\{\left(\mu_A(x_2) - \mu_B(x_2)\right)^2 + \left(\eta_A(x_2) - \eta_B(x_2)\right)^2 + \left(\upsilon_A(x_2) - \upsilon_B(x_2)\right)^2 + \left(\pi_A(x_2) - \pi_B(x_2)\right)^2\right\} + \left\{\left(\mu_A(x_3) - \mu_B(x_3)\right)^2 + \left(\eta_A(x_3) - \eta_B(x_3)\right)^2 + \left(\upsilon_A(x_3) - \upsilon_B(x_3)\right)^2 + \left(\pi_A(x_3) - \pi_B(x_3)\right)^2\right\}\right]\right\}^{\frac{1}{2}}$$

$$= 1 - \left\{\frac{1}{6}[\{(-0.1)^2 + (-0.1)^2 + (0.2)^2 + (0.0)^2\} + \{(-0.3)^2 + (0.1)^2 + (0.2)^2 + (0.0)^2\} + \{(0.2)^2 + (-0.1)^2 + (0.0)^2 + (-0.1)^2\}]\right\}^{\frac{1}{2}} = 1 - \left\{\frac{1}{6}[\{0.01 + 0.01 + 0.04 + 0.0\} + \{0.09 + 0.01 + 0.04 + 0.0\} + \{0.04 + 0.01 + 0.0 + 0.01\}]\right\}^{\frac{1}{2}} = 1 - \left\{\frac{1}{6}[0.06 + 0.14 + 0.06]\right\}^{\frac{1}{2}} = 1 - \{0.04\}^{\frac{1}{2}} = 1 - 0.20 = 0.80$$

From the equation (1.4), we get

$$s_{+\infty}(A,B) = 1 - \max_{1\le j\le n}\left\{\frac{|\mu_A(x_j)-\mu_B(x_j)|+|\eta_A(x_j)-\eta_B(x_j)|+|\upsilon_A(x_j)-\upsilon_B(x_j)|+|\pi_A(x_j)-\pi_B(x_j)|}{2n}\right\}$$

$$= 1 - \max\left\{\begin{array}{l}\frac{|\mu_A(x_1)-\mu_B(x_1)|+|\eta_A(x_1)-\eta_B(x_1)|+|\upsilon_A(x_1)-\upsilon_B(x_1)|+|\pi_A(x_1)-\pi_B(x_1)|}{2\times 3},\\ \frac{|\mu_A(x_2)-\mu_B(x_2)|+|\eta_A(x_2)-\eta_B(x_2)|+|\upsilon_A(x_2)-\upsilon_B(x_2)|+|\pi_A(x_2)-\pi_B(x_2)|}{2\times 3},\\ \frac{|\mu_A(x_3)-\mu_B(x_3)|+|\eta_A(x_3)-\eta_B(x_3)|+|\upsilon_A(x_3)-\upsilon_B(x_3)|+|\pi_A(x_3)-\pi_B(x_3)|}{2\times 3}\end{array}\right\}$$

$$= 1 - max \left\{\frac{0.1+0.1+0.2+0.0}{6}, \frac{0.3+0.1+0.2+0.0}{6}, \frac{0.2+0.1+0.0+0.1}{6}\right\}$$

$$= 1 - max \left\{\frac{0.4}{6}, \frac{0.6}{6}, \frac{0.4}{6}\right\} = 1 - max \{0.07, 0.10, 0.07\} = 1 - 0.10 = 0.90$$

From the equation (1.2w), we get

$$\bar{s}_1(A,B) = 1 - \frac{1}{2n}\sum_{j=1}^{n}\omega_j\left[\left|\mu_A(x_j) - \mu_B(x_j)\right| + \left|\eta_A(x_j) - \eta_B(x_j)\right| + \left|\upsilon_A(x_j) - \upsilon_B(x_j)\right| + \left|\pi_A(x_j) - \pi_B(x_j)\right|\right] = 1 - \frac{1}{2\times 3}\,[\omega_1\{|\mu_A(x_1) - \mu_B(x_1)| + |\eta_A(x_1) - \eta_B(x_1)| + |\upsilon_A(x_1) - \upsilon_B(x_1)| + |\pi_A(x_1) - \pi_B(x_1)|\} + \omega_2\{|\mu_A(x_2) - \mu_B(x_2)| + |\eta_A(x_2) - \eta_B(x_2)| + |\upsilon_A(x_2) - \upsilon_B(x_2)| + |\pi_A(x_2) - \pi_B(x_2)|\} + \omega_3\{|\mu_A(x_3) - \mu_B(x_3)| + |\eta_A(x_3) - \eta_B(x_3)| + |\upsilon_A(x_3) - \upsilon_B(x_3)| + |\pi_A(x_3) - \pi_B(x_3)|\}]$$

$$= 1 - \frac{1}{6}\,[0.5\{0.1 + 0.1 + 0.2 + 0.0\} + 0.2\{0.3 + 0.1 + 0.2 + 0.0\} + 0.3\{0.2 + 0.1 + 0.0 + 0.1\}] = 1 - \frac{1}{6}\,[0.5\{0.4\} + 0.2\{0.6\} + 0.3\{0.4\}] = 1 - \frac{1}{6}\,[0.20 + 0.12 + 0.12] = 1 - 0.07 = 0.93$$

From the equation (1.3w), we get

$$\bar{s}_2(A,B) = 1 - \sqrt{\frac{1}{2n}\sum_{j=1}^{n}\omega_j\left[\left(\mu_A(x_j) - \mu_B(x_j)\right)^2 + \left(\eta_A(x_j) - \eta_B(x_j)\right)^2 + \left(\upsilon_A(x_j) - \upsilon_B(x_j)\right)^2 + \left(\pi_A(x_j) - \pi_B(x_j)\right)^2\right]}$$

$$= 1 - \left\{\frac{1}{2n}\sum_{j=1}^{n}\omega_j\left[\left(\mu_A(x_j) - \mu_B(x_j)\right)^2 + \left(\eta_A(x_j) - \eta_B(x_j)\right)^2 + \left(\upsilon_A(x_j) - \upsilon_B(x_j)\right)^2 + \left(\pi_A(x_j) - \pi_B(x_j)\right)^2\right]\right\}^{\frac{1}{2}} = 1 - \left\{\frac{1}{2\times 3}\left\{\omega_1\left[\left(\mu_A(x_1) - \mu_B(x_1)\right)^2 + \left(\eta_A(x_1) - \eta_B(x_1)\right)^2 + \left(\upsilon_A(x_1) - \upsilon_B(x_1)\right)^2 + \left(\pi_A(x_1) - \pi_B(x_1)\right)^2\right] + \omega_2\left[\left(\mu_A(x_2) - \mu_B(x_2)\right)^2 + \left(\eta_A(x_1) - \eta_B(x_2)\right)^2 + \left(\upsilon_A(x_2) - \upsilon_B(x_2)\right)^2 + \left(\pi_A(x_2) - \pi_B(x_2)\right)^2\right] + \omega_3\left[\left(\mu_A(x_3) - \mu_B(x_3)\right)^2 + \left(\eta_A(x_3) - \eta_B(x_3)\right)^2 + \left(\upsilon_A(x_3) - \upsilon_B(x_3)\right)^2 + \left(\pi_A(x_3) - \pi_B(x_3)\right)^2\right]\right\}\right\}^{\frac{1}{2}}$$

$$= 1 - \left\{\frac{1}{6}\,\{0.5[0.01 + 0.01 + 0.04 + 0.0] + 0.2[0.09 + 0.01 + 0.04 + 0.0] + 0.3[0.04 + 0.01 + 0.0 + 0.01]\}\right\}^{\frac{1}{2}} = 1 - \left\{\frac{1}{6}\,\{0.5[0.6] + 0.2[0.14] + 0.3[0.06]\}\right\}^{\frac{1}{2}} = 1 - \left\{\frac{1}{6}\,\{0.30 + 0.03 + 0.02\}\right\}^{\frac{1}{2}} = 1 - \{0.06\}^{\frac{1}{2}} = 1 - 0.24 = 0.76$$

From the equation (1.4w), we get

$$\bar{s}_{+\infty}(A,B) = 1 - \max_{1 \le j \le n}\left\{\frac{\omega_j\left[|\mu_A(x_j) - \mu_B(x_j)| + |\eta_A(x_j) - \eta_B(x_j)| + |\upsilon_A(x_j) - \upsilon_B(x_j)| + |\pi_A(x_j) - \pi_B(x_j)|\right]}{2n}\right\}$$

$$= 1 - max\left\{\begin{array}{l}\frac{\omega_1[|\mu_A(x_1) - \mu_B(x_1)| + |\eta_A(x_1) - \eta_B(x_1)| + |\upsilon_A(x_1) - \upsilon_B(x_1)| + |\pi_A(x_1) - \pi_B(x_1)|]}{2\times 3},\\ \frac{\omega_2[|\mu_A(x_2) - \mu_B(x_2)| + |\eta_A(x_2) - \eta_B(x_2)| + |\upsilon_A(x_2) - \upsilon_B(x_2)| + |\pi_A(x_2) - \pi_B(x_2)|]}{2\times 3},\\ \frac{\omega_3[|\mu_A(x_3) - \mu_B(x_3)| + |\eta_A(x_3) - \eta_B(x_3)| + |\upsilon_A(x_3) - \upsilon_B(x_3)| + |\pi_A(x_3) - \pi_B(x_3)|]}{2\times 3}\end{array}\right\}$$

$= 1 - max \left\{ \frac{0.5[0.1+0.1+0.2+0.0]}{6}, \frac{0.2[0.3+0.1+0.2+0.0]}{6}, \frac{0.3[0.2+0.1+0.0+0.1]}{6} \right\} = 1 - max \left\{ \frac{0.5[0.4]}{6}, \frac{0.2[0.6]}{6}, \frac{0.3[0.4]}{6} \right\}$

$= 1 - max \quad \{0.03, 0.02, 0.02\} = 1 - 0.03 = 0.97$

Theorem 3.2: If $s(A,B)$ is the similarity degree between two PFSs A and B, then $d(A,B) = 1 - s(A,B)$ is the normalized distance between A and B.

Proof: Trivial.

Theorem 3.3: If $d(A,B)$ is the normalized distance between two PFSs A and B, then $s(A,B) = 1 - d(A,B)$ is the similarity degree between A and B.

Proof: Trivial.

Using the **Theorem 3.2** and equation (1.1), the Minkowski normalized distance between A and B is

$$d_q(A,B) = \sqrt[q]{\frac{1}{2n} \sum_{j=1}^{n} \left[\left(\mu_A(x_j) - \mu_B(x_j)\right)^q + \left(\eta_A(x_j) - \eta_B(x_j)\right)^q + \left(\upsilon_A(x_j) - \upsilon_B(x_j)\right)^q + \left(\pi_A(x_j) - \pi_B(x_j)\right)^q \right]} \quad (2.1)$$

From the equation (1.2), the Hamming normalized distance between A and B is

$$d_1(A,B) = \frac{1}{2n} \sum_{j=1}^{n} \left[\left|\mu_A(x_j) - \mu_B(x_j)\right| + \left|\eta_A(x_j) - \eta_B(x_j)\right| + \left|\upsilon_A(x_j) - \upsilon_B(x_j)\right| + \left|\pi_A(x_j) - \pi_B(x_j)\right| \right] \quad (2.2)$$

From the equation (1.3), the Euclidean normalized distance between A and B is

$$d_2(A,B) = \sqrt{\frac{1}{2n} \sum_{j=1}^{n} \left[\left(\mu_A(x_j) - \mu_B(x_j)\right)^2 + \left(\eta_A(x_j) - \eta_B(x_j)\right)^2 + \left(\upsilon_A(x_j) - \upsilon_B(x_j)\right)^2 + \left(\pi_A(x_j) - \pi_B(x_j)\right)^2 \right]} \quad (2.3)$$

From the equation (1.4), the Chebyshev normalized distance between A and B is

$$d_{+\infty}(A,B) = \max_{1 \le j \le n} \left\{ \frac{|\mu_A(x_j) - \mu_B(x_j)| + |\eta_A(x_j) - \eta_B(x_j)| + |\upsilon_A(x_j) - \upsilon_B(x_j)| + |\pi_A(x_j) - \pi_B(x_j)|}{2n} \right\} \quad (2.4)$$

Theorem 3.4: $d_q(A,B)$, $d_1(A,B)$, $d_2(A,B)$ and $d_{+\infty}(A,B)$ defined by equations (2.1) – (2.4) are the normalized distances between the PFSs A and B respectively.

Now, the weighted Minkowski normalized distance between A and B is

$$\bar{d}_q(A,B) = \sqrt[q]{\frac{1}{2n} \sum_{j=1}^{n} \omega_j \left[\left(\mu_A(x_j) - \mu_B(x_j)\right)^q + \left(\eta_A(x_j) - \eta_B(x_j)\right)^q + \left(\upsilon_A(x_j) - \upsilon_B(x_j)\right)^q + \left(\pi_A(x_j) - \pi_B(x_j)\right)^q \right]} \quad (2.1w)$$

The weighted Hamming normalized distance between A and B is

$$\bar{d}_1(A,B) = \frac{1}{2n} \sum_{j=1}^{n} \omega_j \left[\left|\mu_A(x_j) - \mu_B(x_j)\right| + \left|\eta_A(x_j) - \eta_B(x_j)\right| + \left|\upsilon_A(x_j) - \upsilon_B(x_j)\right| + \left|\pi_A(x_j) - \pi_B(x_j)\right| \right] \quad (2.2w)$$

The weighted Euclidean normalized distance between A and B is

$\bar{d}_2(A,B) =$

$$\sqrt{\frac{1}{2n}\sum_{j=1}^{n}\omega_j\left[\left(\mu_A(x_j)-\mu_B(x_j)\right)^2+\left(\eta_A(x_j)-\eta_B(x_j)\right)^2+\left(\upsilon_A(x_j)-\upsilon_B(x_j)\right)^2+\left(\pi_A(x_j)-\pi_B(x_j)\right)^2\right]}$$

(2.3w)

The weighted Chebyshev normalized distance between A and B is

$$\bar{d}_{+\infty}(A,B)=\max_{1\le j\le n}\left\{\frac{\omega_j(|\mu_A(x_j)-\mu_B(x_j)|+|\eta_A(x_j)-\eta_B(x_j)|+|\upsilon_A(x_j)-\upsilon_B(x_j)|+|\pi_A(x_j)-\pi_B(x_j)|)}{2n}\right\}$$

(2.4w)

Example 2: The PFSs A and B and the weights of the elements x_1, x_2 and x_3 are given as in **Example 1**.

From the equation (2.2), we get

$d_1(A,B)=\frac{1}{2n}\sum_{j=1}^{n}\left[\left|\mu_A(x_j)-\mu_B(x_j)\right|+\left|\eta_A(x_j)-\eta_B(x_j)\right|+\left|\upsilon_A(x_j)-\upsilon_B(x_j)\right|+\left|\pi_A(x_j)-\pi_B(x_j)\right|\right]$

$=\frac{1}{2\times3}[(|\mu_A(x_1)-\mu_B(x_1)|+|\eta_A(x_1)-\eta_B(x_1)|+|\upsilon_A(x_1)-\upsilon_B(x_1)|+|\pi_A(x_1)-\pi_B(x_1)|)+(|\mu_A(x_2)-\mu_B(x_2)|+|\eta_A(x_2)-\eta_B(x_2)|+|\upsilon_A(x_2)-\upsilon_B(x_2)|+|\pi_A(x_2)-\pi_B(x_2)|)+(|\mu_A(x_3)-\mu_B(x_3)|+|\eta_A(x_3)-\eta_B(x_3)|+|\upsilon_A(x_3)-\upsilon_B(x_3)|+|\pi_A(x_3)-\pi_B(x_3)|)]$

$=\frac{1}{6}[(|0.5-0.6|+|0.2-0.3|+|0.3-0.1|+|0.0-0.0|)+(|0.2-0.5|+|0.3-0.2|+|0.4-0.2|+|0.1-0.1|)+(|0.6-0.4|+|0.3-0.4|+|0.1-0.1|+|0.0-0.1|)]=\frac{1}{6}[(0.1+0.1+0.2+0.0)+(0.3+0.1+0.2+0.0)+(0.2+0.1+0.0+0.1)]=\frac{1}{6}[(0.1+0.1+0.2+0.0)+(0.3+0.1+0.2+0.0)+(0.2+0.1+0.0+0.1)]=\frac{1}{6}[0.4+0.6+0.4]=\frac{1.4}{6}=0.23$

From the equation (2.3), we get

$$d_2(A,B)=\sqrt{\frac{1}{2n}\sum_{j=1}^{n}\left[\left(\mu_A(x_j)-\mu_B(x_j)\right)^2+\left(\eta_A(x_j)-\eta_B(x_j)\right)^2+\left(\upsilon_A(x_j)-\upsilon_B(x_j)\right)^2+\left(\pi_A(x_j)-\pi_B(x_j)\right)^2\right]}$$

$=\left\{\frac{1}{2\times3}\left[\left\{\left(\mu_A(x_1)-\mu_B(x_1)\right)^2+\left(\eta_A(x_1)-\eta_B(x_1)\right)^2+\left(\upsilon_A(x_1)-\upsilon_B(x_1)\right)^2+\left(\pi_A(x_1)-\pi_B(x_1)\right)^2\right\}+\left\{\left(\mu_A(x_2)-\mu_B(x_2)\right)^2+\left(\eta_A(x_2)-\eta_B(x_2)\right)^2+\left(\upsilon_A(x_2)-\upsilon_B(x_2)\right)^2+\left(\pi_A(x_2)-\pi_B(x_2)\right)^2\right\}+\left\{\left(\mu_A(x_3)-\mu_B(x_3)\right)^2+\left(\eta_A(x_3)-\eta_B(x_3)\right)^2+\left(\upsilon_A(x_3)-\upsilon_B(x_3)\right)^2+\left(\pi_A(x_3)-\pi_B(x_3)\right)^2\right\}\right]\right\}^{\frac{1}{2}}=\left\{\frac{1}{6}[\{(-0.1)^2+(-0.1)^2+(0.2)^2+(0.0)^2\}+\{(-0.3)^2+(0.1)^2+(0.2)^2+(0.0)^2\}+\{(0.2)^2+(-0.1)^2+(0.0)^2+(-0.1)^2\}]\right\}^{\frac{1}{2}}=\left\{\frac{1}{6}[\{0.01+0.01+0.04+0.0\}+\{0.09+0.01+0.04+0.0\}+\{0.04+0.01+0.0+0.01\}]\right\}^{\frac{1}{2}}=\left\{\frac{1}{6}[0.06+0.14+0.06]\right\}^{\frac{1}{2}}=\{0.04\}^{\frac{1}{2}}=0.20$

From the equation (2.4), we get

$$d_{+\infty}(A,B)=\max_{1\le j\le n}\left\{\frac{|\mu_A(x_j)-\mu_B(x_j)|+|\eta_A(x_j)-\eta_B(x_j)|+|\upsilon_A(x_j)-\upsilon_B(x_j)|+|\pi_A(x_j)-\pi_B(x_j)|}{2n}\right\}$$

$$= max \left\{ \begin{array}{c} \frac{|\mu_A(x_1)-\mu_B(x_1)|+|\eta_A(x_1)-\eta_B(x_1)|+|\upsilon_A(x_1)-\upsilon_B(x_1)|+|\pi_A(x_1)-\pi_B(x_1)|}{2\times 3}, \\ \frac{|\mu_A(x_2)-\mu_B(x_2)|+|\eta_A(x_2)-\eta_B(x_2)|+|\upsilon_A(x_2)-\upsilon_B(x_2)|+|\pi_A(x_2)-\pi_B(x_2)|}{2\times 3}, \\ \frac{|\mu_A(x_3)-\mu_B(x_3)|+|\eta_A(x_3)-\eta_B(x_3)|+|\upsilon_A(x_3)-\upsilon_B(x_3)|+|\pi_A(x_3)-\pi_B(x_3)|}{2\times 3} \end{array} \right\}$$

$$= max \left\{\frac{0.1+0.1+0.2+0.0}{6}, \frac{0.3+0.1+0.2+0.0}{6}, \frac{0.2+0.1+0.0+0.1}{6}\right\} = max \left\{\frac{0.4}{6}, \frac{0.6}{6}, \frac{0.4}{6}\right\}$$

$$= max \{0.07, 0.10, 0.07\} = 0.10$$

From the equation (2.2w), we get

$$\bar{d}_1(A,B) = \frac{1}{2n}\sum_{j=1}^{n}\omega_j\left[\left|\mu_A(x_j)-\mu_B(x_j)\right| + \left|\eta_A(x_j)-\eta_B(x_j)\right| + \left|\upsilon_A(x_j)-\upsilon_B(x_j)\right| + \left|\pi_A(x_j)-\pi_B(x_j)\right|\right]$$

$$= \frac{1}{2\times 3}\,[\omega_1\{|\mu_A(x_1)-\mu_B(x_1)| + |\eta_A(x_1)-\eta_B(x_1)| + |\upsilon_A(x_1)-\upsilon_B(x_1)| + |\pi_A(x_1)-\pi_B(x_1)|\} + \omega_2\{|\mu_A(x_2)-\mu_B(x_2)| + |\eta_A(x_2)-\eta_B(x_2)| + |\upsilon_A(x_2)-\upsilon_B(x_2)| + |\pi_A(x_2)-\pi_B(x_2)|\} + \omega_3\{|\mu_A(x_3)-\mu_B(x_3)| + |\eta_A(x_3)-\eta_B(x_3)| + |\upsilon_A(x_3)-\upsilon_B(x_3)| + |\pi_A(x_3)-\pi_B(x_3)|\}] = \frac{1}{6}\,[0.5\{0.1+0.1+0.2+0.0\} + 0.2\{0.3+0.1+0.2+0.0\} + 0.3\{0.2+0.1+0.0+0.1\}] = \frac{1}{6}\,[0.5\{0.4\} + 0.2\{0.6\} + 0.3\{0.4\}] = \frac{1}{6}\,[0.20+0.12+0.12] = 0.07$$

From the equation (2.3w), we get

$$\bar{d}_2(A,B) = \sqrt{\frac{1}{2n}\sum_{j=1}^{n}\omega_j\left[\left(\mu_A(x_j)-\mu_B(x_j)\right)^2 + \left(\eta_A(x_j)-\eta_B(x_j)\right)^2 + \left(\upsilon_A(x_j)-\upsilon_B(x_j)\right)^2 + \left(\pi_A(x_j)-\pi_B(x_j)\right)^2\right]}$$

$$= \left\{\frac{1}{2n}\sum_{j=1}^{n}\omega_j\left[\left(\mu_A(x_j)-\mu_B(x_j)\right)^2 + \left(\eta_A(x_j)-\eta_B(x_j)\right)^2 + \left(\upsilon_A(x_j)-\upsilon_B(x_j)\right)^2 + \left(\pi_A(x_j)-\pi_B(x_j)\right)^2\right]\right\}^{\frac{1}{2}} = \left\{\frac{1}{2\times 3}\left\{\omega_1\left[\left(\mu_A(x_1)-\mu_B(x_1)\right)^2 + \left(\eta_A(x_1)-\eta_B(x_1)\right)^2 + \left(\upsilon_A(x_1)-\upsilon_B(x_1)\right)^2 + \left(\pi_A(x_1)-\pi_B(x_1)\right)^2\right] + \omega_2\left[\left(\mu_A(x_2)-\mu_B(x_2)\right)^2 + \left(\eta_A(x_1)-\eta_B(x_2)\right)^2 + \left(\upsilon_A(x_2)-\upsilon_B(x_2)\right)^2 + \left(\pi_A(x_2)-\pi_B(x_2)\right)^2\right] + \omega_3\left[\left(\mu_A(x_3)-\mu_B(x_3)\right)^2 + \left(\eta_A(x_3)-\eta_B(x_3)\right)^2 + \left(\upsilon_A(x_3)-\upsilon_B(x_3)\right)^2 + \left(\pi_A(x_3)-\pi_B(x_3)\right)^2\right]\right\}\right\}^{\frac{1}{2}}$$

$$= \left\{\frac{1}{6}\,\{0.5[0.01+0.01+0.04+0.0] + 0.2[0.09+0.01+0.04+0.0] + 0.3[0.04+0.01+0.0+0.01]\}\right\}^{\frac{1}{2}} = \left\{\frac{1}{6}\,\{0.5[0.6] + 0.2[0.14] + 0.3[0.06]\}\right\}^{\frac{1}{2}} = \left\{\frac{1}{6}\,\{0.30+0.03+0.02\}\right\}^{\frac{1}{2}} = \{0.06\}^{\frac{1}{2}} = 0.24$$

From the equation (2.4w), we get

$$\bar{d}_{+\infty}(A,B) = \max_{1\le j\le n}\left\{\frac{\omega_j[|\mu_A(x_j)-\mu_B(x_j)|+|\eta_A(x_j)-\eta_B(x_j)|+|\upsilon_A(x_j)-\upsilon_B(x_j)|+|\pi_A(x_j)-\pi_B(x_j)|]}{2n}\right\}$$

$$= max \left\{ \begin{array}{c} \frac{\omega_1[|\mu_A(x_1)-\mu_B(x_1)|+|\eta_A(x_1)-\eta_B(x_1)|+|\upsilon_A(x_1)-\upsilon_B(x_1)|+|\pi_A(x_1)-\pi_B(x_1)|]}{2\times 3}, \\ \frac{\omega_2[|\mu_A(x_2)-\mu_B(x_2)|+|\eta_A(x_2)-\eta_B(x_2)|+|\upsilon_A(x_2)-\upsilon_B(x_2)|+|\pi_A(x_2)-\pi_B(x_2)|]}{2\times 3}, \\ \frac{\omega_3[|\mu_A(x_3)-\mu_B(x_3)|+|\eta_A(x_3)-\eta_B(x_3)|+|\upsilon_A(x_3)-\upsilon_B(x_3)|+|\pi_A(x_3)-\pi_B(x_3)|]}{2\times 3} \end{array} \right\}$$

$= max \left\{\frac{0.5[0.1+0.1+0.2+0.0]}{6}, \frac{0.2[0.3+0.1+0.2+0.0]}{6}, \frac{0.3[0.2+0.1+0.0+0.1]}{6}\right\}$

$= max \left\{\frac{0.5[0.4]}{6}, \frac{0.2[0.6]}{6}, \frac{0.3[0.4]}{6}\right\} =$
$max \{0.03, 0.02, 0.02\} = 0.03$

It is easy to see from **Examples 1 and 2** that,

$d_1(A,B) + s_1(A,B) = 1$, $d_2(A,B) + s_2(A,B) = 1$, $d_{+\infty}(A,B) + s_{+\infty}(A,B) = 1$,

$\bar{d}_1(A,B) + \bar{s}_1(A,B) = 1$, $\bar{d}_2(A,B) + \bar{s}_2(A,B) = 1$ and $\bar{d}_{+\infty}(A,B) + \bar{s}_{+\infty}(A,B) = 1$, which are just the examples for the **Theorems 3.1 and 3.2.**

4 A New Method for Similarity Measures between two Picture Fuzzy Sets

Definition 4.1: Let $X = \{x_1, x_2, \cdots\cdots\cdots, x_n\}$ be a finite universe of discourse. For each PFS $A = \{(x_i, \mu_A(x_i), \eta_A(x_i), \upsilon_A(x_i)) : x_i \in X\}$, let

$\mu_A(x_i) = \mu_A(x_i) + \frac{\mu_A(x_i)+\eta_A(x_i)+(1-\nu_A(x_i))}{4} \times \pi_A(x_i)$, $\eta_A(x_i) = \eta_A(x_i)$ and $\nu_A(x_i) = \nu_A(x_i)$. $(*)$

Here the above formula will calculate the modified positive membership degree $\mu_A(x_i)$ which is influenced by the abstain and refusal groups i,e $\frac{\mu_A(x_i)+\eta_A(x_i)+(1-\nu_A(x_i))}{4} \times \pi_A(x_i)$ indicates the possibility that the abstain and refusal groups tend to support positive membership.

Definition 4.2: In the following, we give a new definition for a similarity measure between two PFSs. Let $A = \{(x_i, \mu_A(x_i), \eta_A(x_i), \upsilon_A(x_i)) : x_i \in X\}$ and $B = \{(x_i, \mu_B(x_i), \eta_B(x_i), \upsilon_B(x_i)) : x_i \in X\}$ be two PFSs. Then the similarity measure between A and B is given by:

$$s(A,B) = 1 - \frac{1}{n^{\frac{1}{\alpha}}} \left[\sum_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)|^\alpha + |\eta_A(x_i) - \eta_B(x_i)|^\alpha + |\upsilon_A(x_i) - \upsilon_B(x_i)|^\alpha\right]^{\frac{1}{\alpha}} ; \alpha > 0$$

If let $\alpha = 1$, then the above equation is reduced to the following formula:

$$s(A,B) = 1 - \frac{1}{n} \left[\sum_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)| + |\eta_A(x_i) - \eta_B(x_i)| + |\upsilon_A(x_i) - \upsilon_B(x_i)|\right] \quad (**)$$

5 Algorithm and Applications

5.1. Algorithm for Pattern Recognition

Let $X = \{x_1, x_2, \cdots\cdots\cdots, x_n\}$ be a finite universe of discourse. Suppose that there exist m patterns represented by the PFSs $P_j = \{(x_j, \mu_{P_j}(x_i), \eta_{P_j}(x_i), \upsilon_{P_j}(x_i)) : x_i \in X\}$ $(j = 1,2,\cdots\cdots, m)$ and a test sample $D = \{(x_j, \mu_d(x_i), \eta_d(x_i), \upsilon_d(x_i)) : x_i \in X\}$. Which pattern does the sample D belong to? The recognition steps are as follows:

Step 1: Calculate the similarity measure $s(P_j, D)(j = 1,2,\cdots\cdots, m)$ between P_j and D by the equation $(**)$**.**

Step 2: Pick up the maximum value from $s(P_j, D)$ $(j = 1,2,\cdots\cdots, m)$ and denoted by $s(P_{jo}, D)$. i,e $s(P_{jo}, D) = max_{1\le j\le m}\{s(P_j, D)\}$

Then the test sample D is classified to pattern P_{jo} according to the principle of the maximum of the similarity measure. i,e the sample D belongs to the pattern P_{jo} .

5.2 Application of the Picture Fuzzy Sets in Pattern Recognition

Example (Dutta, 2017): Suppose the set of patients $P = \{P_1, P_2, P_3, P_4\}$ and the set of symptoms $S = \{S_1 = \text{Temperature}, S_2 = \text{Headache}, S_3 = \text{Stomach pain}, S_4 = \text{Cough}, S_5 = \text{Chest pain}\}$.The set of diagnoses is $D = \{D_1 = \text{Viral Fever}, D_2 = \text{Malaria}, D_3 = \text{Typhoid}, D_4 = \text{Stomach problem}, D_5 = \text{Chest problem}\}$.

Table-1 represents the symptoms of the patients and **Table-2** represents the symptoms of the diseases and the tables are carried in the form of picture fuzzy information. The proposed similarity measure equation $(**)$ is used to make a proper diagnosis for each patient. As per the principle of the maximum of similarity measures, the greater similarity measure indicates a proper diagnosis.

Table-1: Symptoms Characteristic for the Patients.

	S_1	S_2	S_3	S_4	S_5
P_1	(0.80,0.00,0.10)	(0.60,0.30,0.10)	(0.20,0.40,0.40)	(0.60,0.15,0.10)	(0.10,0.40,0.40)
P_2	(0.00,0.50,0.40)	(0.40,0.25,0.30)	(0.60,0.20,0.10)	(0.10,0.30,0.60)	(0.10,0.35,0.40)
P_3	(0.80,0.00,0.10)	(0.80,0.00,0.10)	(0.00,0.40,0.50)	(0.20,0.30,0.40)	(0.00,0.40,0.40)
P_4	(0.60,0.20,0.10)	(0.50,0.25,0.25)	(0.30,0.30,0.20)	(0.70,0.00,0.25)	(0.30,0.40,0.20)

Table-2: Symptoms Characteristic for the Diagnoses.

	S_1	S_2	S_3	S_4	S_5
D_1	(0.40,0.00,0.00)	(0.30,0.20,0.40)	(0.10,0.35,0.50)	(0.40,0.30,0.20)	(0.10,0.25,0.50)
D_2	(0.70,0.00,0.00)	(0.20,0.40,0.35)	(0.00,0.40,0.50)	(0.70,0.10,0.00)	(0.10,0.30,0.50)
D_3	(0.30,0.40,0.30)	(0.60,0.20,0.10)	(0.20,0.30,0.40)	(0.20,0.35,0.30)	(0.10,0.20,0.60)
D_4	(0.10,0.30,0.50)	(0.20,0.40,0.30)	(0.80,0.00,0.00)	(0.20,0.40,0.30)	(0.20,0.35,0.30)
D_5	(0.10,0.30,0.50)	(0.00,0.50,0.35)	(0.20,0.30,0.50)	(0.20,0.35,0.40)	(0.80,0.00,0.10)

According to the formula $(*)$, **Table-3** and **Table-4** will be formed by using the data in **Table-1** and **Table-2** respectively.

Table-3: Symptoms Characteristic for the Patients.

	S_1	S_2	S_3	S_4	S_5
P_1	(0.84,0.00,0.10)	(0.60,0.30,0.10)	(0.20,0.40,0.40)	(0.66,0.15,0.10)	(0.13,0.40,0.40)
P_2	(0.03,0.50,0.40)	(0.42,0.25,0.30)	(0.64,0.20,0.10)	(0.10,0.30,0.60)	(0.14,0.35,0.40)
P_3	(0.84,0.00,0.10)	(0.84,0.00,0.10)	(0.02,0.40,0.50)	(0.23,0.30,0.40)	(0.05,0.40,0.40)
P_4	(0.64,0.20,0.10)	(0.50,0.25,0.25)	(0.37,0.30,0.20)	(0.72,0.00,0.25)	(0.34,0.40,0.20)

Table-4: Symptoms Characteristic for the Diagnoses.

	S_1	S_2	S_3	S_4	S_5
D_1	(0.61,0.00,0.00)	(0.33,0.20,0.40)	(0.11,0.35,0.50)	(0.44,0.30,0.20)	(0.13,0.25,0.50)
D_2	(0.83,0.00,0.00)	(0.22,0.40,0.35)	(0.02,0.40,0.50)	(0.79,0.10,0.00)	(0.12,0.30,0.50)
D_3	(0.30,0.40,0.30)	(0.64,0.20,0.10)	(0.23,0.30,0.40)	(0.25,0.35,0.30)	(0.12,0.20,0.60)
D_4	(0.12,0.30,0.50)	(0.23,0.40,0.30)	(0.89,0.00,0.00)	(0.23,0.40,0.30)	(0.25,0.35,0.30)
D_5	(0.12,0.30,0.50)	(0.04,0.50,0.35)	(0.20,0.30,0.50)	(0.21,0.35,0.40)	(0.84,0.00,0.10)

According to the formula $(**)$, **Table-5** will be formed by using the data in **Table-3** and **Table-4** respectively.

Table-5: Similarities of Symptoms for each Patient to the set of Possible Diagnoses.

	D_1	D_2	D_3	D_4	D_5
P_1	0.608	**0.678**	0.474	0.054	0.002
P_2	0.250	0.004	0.476	**0.596**	0.256
P_3	**0.556**	0.438	0.482	0.010	0.038
P_4	**0.480**	0.438	0.378	0.262	0.138

For example, we can get $s(P_1, D_1)$ by$(**)$:

$$s(P_1, D_1) = 1 - \frac{1}{5}\left[\sum_{i=1}^{n} |\mu_A(x_i) - \mu_B(x_i)| + |\eta_A(x_i) - \eta_B(x_i)| + |\upsilon_A(x_i) - \upsilon_B(x_i)|\right]$$

$$= 1 - \frac{1}{5}\,[(|0.84 - 0.61| + |0.00 - 0.00| + |0.10 - 0.00|) + (|0.60 - 0.33| + |0.30 - 0.20| + |0.10 - 0.40|) + (|0.20 - 0.11| + |0.40 - 0.35| + |0.40 - 0.50|) + (|0.66 - 0.44| + |0.15 - 0.30| + |0.10 - 0.20|) + (|0.13 - 0.13| + |0.40 - 0.25| + |0.40 - 0.50|)] = 1 - \frac{1}{5}\,[0.33 + 0.67 + 0.24 + 0.47 + 0.25]$$

$$= 1 - \frac{1}{5}\,[1.96] = 1 - 0.392 = 0.608$$

Table-6: The Comparison of all the Results.

	P_1	P_2	P_3	P_4
The Result in Dutta (2017)	Malaria	Stomach problems	Typhoid	Viral Fever
Our Result	Malaria	Stomach problems	Viral Fever	Viral Fever

Then the proper diagnosis D_i for the patient P_j is derived according to the biggest numerical value from the obtained similarity measures in **Table-5**.

From **Table-5**, we can see P_1 suffers from Malaria, P_2 from Stomach problems, P_3 from Viral Fever and P_4 from Viral Fever. Compared with the results in **[6],** the diagnoses for P_1, P_2 and P_4 are the same, but the diagnosis for P_3 is different.

6. Conclusions

Picture fuzzy set is a modern concept to handle vague data effectively. The measure of distance and similarity degrees are a pair of powerful concept to identify the degrees of similarity and dissimilarity between the PFSs. In this paper, a new method to measure the similarities between the picture fuzzy sets is developed by considering the influence of the abstain and refusal groups. Finally, the validity and reliability of the proposed similarity measure is illustrated through a pattern recognition in medical diagnosis and compare the result with the results found by other existing methods.

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