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Natural Convective Flow along a Vertical Flat Plate with Sinusoidal Surface Heat Flux Case

Research Article

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Abstract : Boundary-layer natural convective flow of Newtonian fluids on a vertically heated wall is influenced by the existence of the sinusoidal surface heat flux case has been investigated with a mean value constant more than that of the ambient temperature of the fluid. Two numerical techniques have been imposed to solve the problem; firstly implicit finite difference method (IFDM) and secondly direct numerical scheme (DNS). The similarities of the numerical results are outstanding. The simulations are presented for the case of stream lines and isotherms, shear stresses and heat transfer rate in terms of the local skin-friction and local Nusselt number respectively.

Keywords: *Natural convection ● Sinusoidal heat flux ● Stream lines ● Isotherms*

1. Introduction

Steady laminar natural convective flow of Newtonian fluids along a vertical flat plate with sinusoidal surface heat flux plays a vital role in frequent engineering applications such as many transportation procedures in industries. We are focused on the surface variations, which is called sinusoidal variations, with an average temperature more than that of the ambient fluid temperature. The aim of our work to simplify the problem by considering a surface heat flux to examine the solid conduction effect with accurate analysis. Therefore, we may find a lots of data using implicit finite difference method (IFDM) and direct numerical scheme (DNS) numerical methods.

Many investigators have been investigated on the surface variations effect. Yao (1983) and Moulic and Yao (1989a; b) have studied the streamwise surface undulations effects of natural and mixed convections from vertical surfaces with uniform temperatures. Moreover, Kim (1997) and Chiu and Chou (1993) have elaborated the analyses to micro-polar fluids, magneto-hydrodynamic convection and non-Newtonian convection. Rees and Pop (1994a, b; 1995a, b; 1997) have been studied a big variation of similar flows in the porous media. The steady streamwise surface temperature differences has been considered by Rees (1999) on vertical natural convection. Roy and Hossain (2010) have also considered the mixed effects of species concentration variations and streamwise temperature.

The current study shows that how the sinusoidal surface heat flux profiles in the direction of streamwise modify the different similar flow. In this paper we have attempted this subject with two numerical methods. Results are submitted in terms of the heat transfer rate and shear stresses. Also stream lines and isotherms are given completely.

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2. Problem formulations

Laminar two-dimensional viscous and incompressible steady natural convective flow from a vertical flat plate is considered. Consider the surface heat flux of the plate is $q_w(x)$. The physical formation is illustrated in Fig. 1.

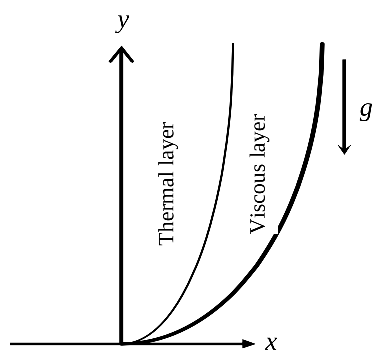

Figure 1: Physical model with coordinating system.

The governing equations with the Boussinesq approximation are:

$$\frac{\partial \hat{u}}{\partial \hat{x}}+\frac{\partial \hat{v}}{\partial \hat{y}}=0, \tag{1}$$

$$\rho\left(\hat{u}\frac{\partial \hat{u}}{\partial \hat{x}}+\hat{v}\frac{\partial \hat{u}}{\partial \hat{y}}\right)=\mu\frac{\partial^2 \hat{u}}{\partial \hat{y}^2}+\rho g\beta\left(T-T_\infty\right), \tag{2}$$

$$\hat{u}\frac{\partial T}{\partial \hat{x}}+\hat{v}\frac{\partial T}{\partial \hat{y}}=\frac{k}{\rho C_p}\frac{\partial^2 T}{\partial \hat{y}^2}. \tag{3}$$

Here, the velocity components along the axes are $\hat{u}$ and $\hat{v}$, g is the gravitational force, ρ is the fluid density, k is the thermal conductivity, β is the thermal expansion coefficient, μ is the fluid viscosity, C_p is the specific heat at constant pressure and T is the temperature of fluid.

The suitable boundary conditions to solve equations (1)-(3) are:

$$\hat{u}=\hat{v}=0,\quad -k\frac{\partial T}{\partial \hat{y}}=q_w\left[1+a\sin(\pi\hat{x}/l)\right] \text{ at } \hat{y}=0. \tag{4a}$$

$$\hat{u}\to 0,\quad T\to T_\infty \quad \text{as} \quad \hat{y}\to\infty. \tag{4b}$$

The normalized variables are:

$$x=\frac{\hat{x}}{l},\ y=Gr^{1/5}\left(\frac{\hat{y}}{l}\right),\quad u=\frac{l}{\nu}Gr^{-2/5}\hat{u},$$
$$v=\frac{l}{\nu}Gr^{-1/5}\hat{v},\quad \theta=\frac{T-T_\infty}{\left(\frac{q_w l}{k}\right)}Gr^{1/5},\quad Gr=\frac{g\beta q_w l^4}{\nu^2}. \tag{5}$$

Where, Gr is the Grashof number, ν (=μ/ρ) is the kinematic viscosity and θ is the function of normalized fluid temperature.

Replacing equation (5) into equations (1)-(3), we have the subsequent equations:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{6}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+\theta, \tag{7}$$

$$u\frac{\partial \theta}{\partial x}+v\frac{\partial \theta}{\partial y}=\frac{1}{\Pr}\frac{\partial^2 \theta}{\partial y^2}. \tag{8}$$

The corresponding boundary conditions are:

$$u=v=0,\quad \theta'=-\left[1+a\sin(\pi x)\right] \quad \text{at} \quad y=0, \tag{9a}$$

$$u\to 0,\quad \theta\to 0 \quad \text{as} \quad y\to\infty. \tag{9b}$$

Here Pr is the Prandtl number, defined as

$$\Pr = \frac{\mu C_p}{k}. \tag{10}$$

3. Numerical methods

To investigate the current study we have worked with two numerical techniques, namely, implicit finite difference and direct numerical scheme method, which are described below individually.

3.1 Implicit finite difference method (IFD)

At first, we must reduce the equations (6)-(9) to an appropriate set of equations. For this, we have to propose the subsequent transformations.

$$\psi=x^{4/5}f(x,\eta),\quad \eta=x^{-1/5}y,\quad \theta=x^{1/5}\theta(x,\eta). \tag{11}$$

Here ψ is the non-dimensional stream function defined as

$$u=\frac{\partial \psi}{\partial y},\quad v=-\frac{\partial \psi}{\partial x}. \tag{12}$$

Replace the equations (11) and (12) in the equations (6)-(9) and simplify, we are getting the following transformed equation forms:

$$f'''+\frac{4}{5}ff''-\frac{3}{5}f'^2+\theta=x\left(f'\frac{\partial f'}{\partial x}-f''\frac{\partial f}{\partial x}\right), \tag{13}$$

$$\frac{1}{\Pr}\theta''+\frac{4}{5}f\theta'-\frac{1}{5}f'\theta=x\left(f'\frac{\partial \theta}{\partial x}-\theta'\frac{\partial f}{\partial x}\right). \tag{14}$$

With the boundary conditions:

$$f(x,0)=f'(x,0)=0,\ \theta'(x,0)=-\left[1+a\sin(\pi x)\right], \tag{15a}$$

$$f'(x,\infty)=\theta(x,\infty)=0. \tag{15b}$$

We observed that close to the leading edge ($x\approx 0$), equations (13)-(14) becomes:

$$f'''+\frac{3}{4}ff''-\frac{1}{2}f'^2+\theta=0, \tag{16}$$

$$\frac{1}{\Pr}\theta''+\frac{4}{5}f\theta'-\frac{1}{5}f'\theta=0. \tag{17}$$

With the boundary conditions:

$$f(0) = f'(0) = 0, \quad \theta'(0) = -1, \tag{18a}$$

$$f' \to 0, \quad \theta \to 0 \quad \text{as} \quad y \to \infty . \tag{18b}$$

In the practical applications, the skin-friction coefficient C_f and the Nusselt number Nu are represented in terms of the shearing stress and the rate of heat transfer respectively for the physical quantities of principle interest, which may define as:

$$C_f \left[Gr/4x\right]^{1/5} = f''(x,0), \tag{19}$$

$$Nu\left[Gr/4x\right]^{-1/5} = \frac{1}{\theta(x,0)}. \tag{20}$$

Now we may work with one among many effective and precise implicit finite difference method (the Keller box method), presented by Keller and Cebeci (1971) and describe in more detail in Cebeci and Bradshow (1984), to solve the nonlinear transformed equations (13) and (14).

3.2 Direct numerical scheme (DNS)

Here we have to present the new transformations over the governing equations:

$$X = x, \quad Y = \frac{y}{x^{1/5}}, U = \frac{u}{x^{3/5}}, \quad V = x^{1/5} v. \tag{21}$$

Replace (21) in the equations (6)-(8), we have

$$X\frac{\partial U}{\partial X} + \frac{3}{5}U - \frac{1}{5}Y\frac{\partial U}{\partial Y} + \frac{\partial V}{\partial Y} = 0, \tag{22}$$

$$XU\frac{\partial U}{\partial X} + \left(V - \frac{1}{5}UY\right)\frac{\partial U}{\partial Y} + \frac{3}{5}U^2 = \frac{\partial^2 U}{\partial Y^2} + \theta, \tag{23}$$

$$XU\frac{\partial \theta}{\partial X} + \left(V - \frac{1}{5}UY\right)\frac{\partial \theta}{\partial Y} + \frac{1}{5}U\theta = \frac{1}{\Pr}\frac{\partial^2 \theta}{\partial Y^2}. \tag{24}$$

Along the boundary conditions are

$$U = V = 0, \quad \frac{\partial \theta}{\partial Y} = -\left[1 + a\sin(\pi X)\right] \quad \text{at} \quad Y = 0, \tag{25a}$$

$$U \to 0, \quad \theta \to 0 \quad \text{as} \quad Y \to \infty. \tag{25b}$$

Equations (22)-(25) are discretized for direct numerical scheme (DNS) by central-difference formulae for the diffusion terms and by the forward difference formulae for the convective terms. Therefore, we can find a tri-diagonal equations system, solved by Gaussian elimination method. The solutions getting started from $x = 0.0$, and then forward at the downstream completely. Here $\Delta X = 0.005$ for the X- grid and $\Delta Y = 0.01$ for the Y-grid have been chosen.

The skin-friction coefficient C_f and the Nusselt number Nu may be written as:

$$C_f \left[Gr/4x\right]^{1/5} = \left[\frac{\partial U}{\partial Y}\right]_{Y=0}, \tag{26}$$

$$Nu\left[Gr/4x\right]^{-1/5} = \frac{1}{\theta(x,0)}. \tag{27}$$

4. Results and discussion

4.1 Skin-Friction Coefficient and Nusselt Number

The simulations are presented for the wall shearing stress, $C_f (Gr/4x)^{1/5}$ and heat transfer rate, $Nu(Gr/4x)^{-1/5}$ in terms of the local skin-friction coefficient and the local Nusselt number respectively for the surface temperature wave amplitude, a (= 0.2, 0.4, 0.6, 0.8, 1.0) are obtained for Prandtl number, Pr (= 0.7, 7.0).

The effects of the temperature wave amplitude on the difference of $C_f \left[Gr/4x\right]^{1/5}$ are revealed in Fig. 2 for different Pr (= 0.7, 7.0). The simulations show that the thickness of the fluids is larger at the leading edge than the down edge. For large temperature wave amplitude a, the sinusoidal effects are greater than for small a in both Pr. Also the boundary-layer thickness is higher for Pr = 0.7 than for Pr = 7.0.

From Fig. 5 shows that the thermal boundary-layer effects are more clear and higher for large a.

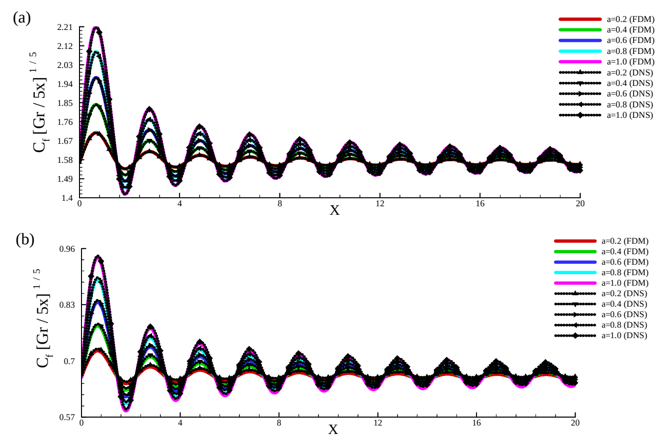

Figure 2: Wall shearing stress for different a: (a) Pr = 0.7, (b) Pr = 7.0.

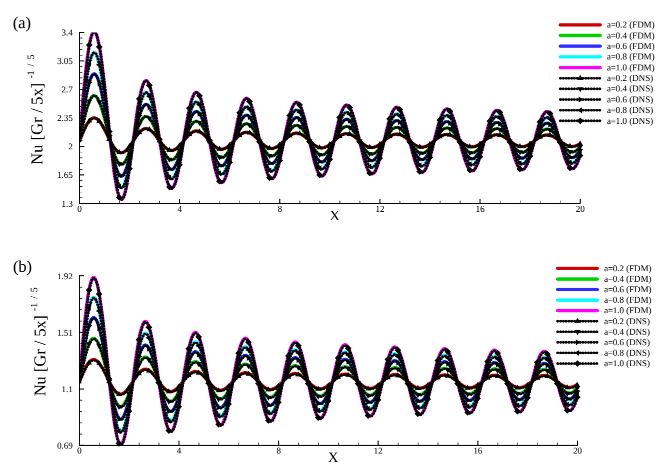

Figure 3: heat transfer rate for different a: (a) Pr = 0.7, (b) Pr = 7.0.

The effects of the same temperature wave amplitude of the surface, *a* on the variation of the heat transfer rate, $Nu(Gr/4X)^{-1/5}$ are revealed in Fig. 3 for different Pr (= 0.7, 7.0). The figures show clearly that at the leading edge the fluid thickness is larger than down edge. For large temperature wave amplitude *a*, the sinusoidal effects are greater than for small *a* in both Pr. Also the thickness of the boundary-layer for Pr = 0.7 is greater than that for Pr = 7.0. By comparing the numerical results of both methods in figures, we show that they are almost same in all regions.

4.2 Stream lines and Isotherms

Figures 4 and 5 reveal the effect of the surface temperature wave amplitude, (*a* = 0.0, 0.2, 0.5, 1.0) for Pr = 0.7 on the development of streamlines and isotherms respectively. Figure 4 shows that streamlines are larger for large *a* (= 1.0) than that for small *a* (= 0.2).

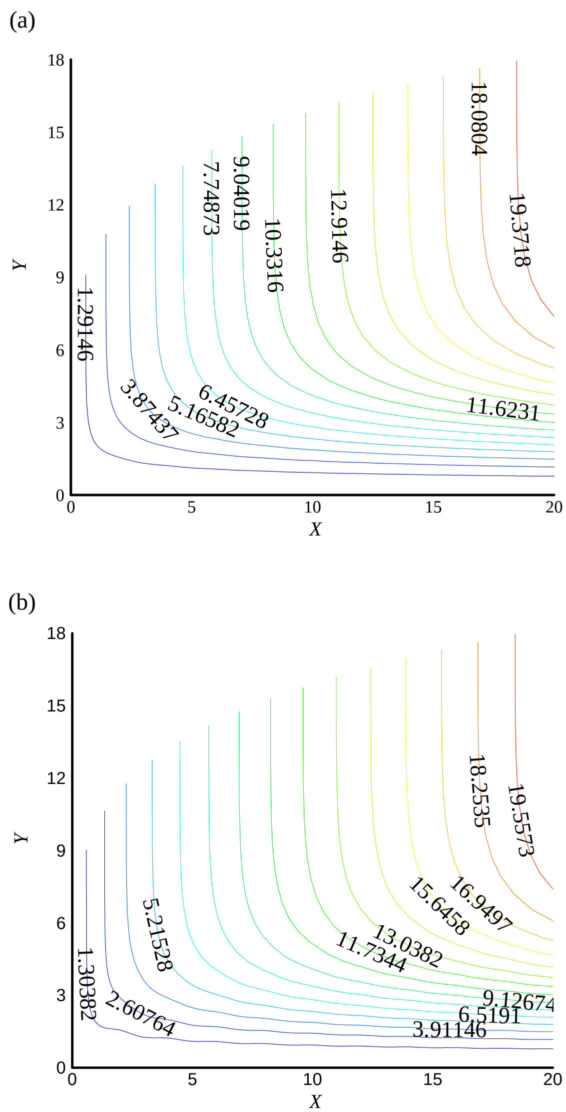

Figure 4: Stream Lines for Pr = 0.7 at (a) *a* = 0.2 and (b) *a* = 1.0.

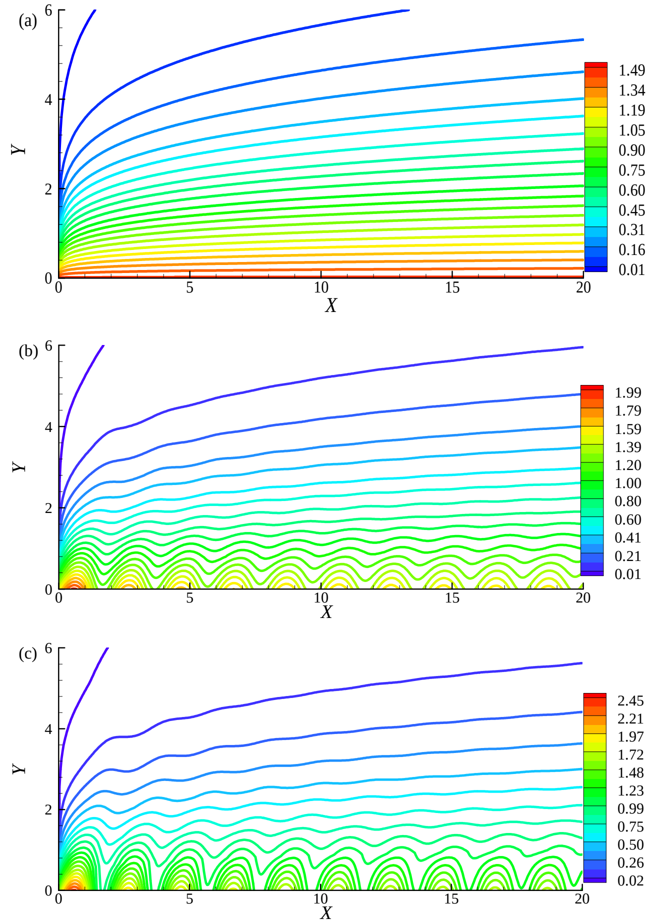

Figure 5: Isotherms for (a) *a* = 0.0, (b) *a* = 0.5 and (c) *a* = 1.0 while Pr = 0.7.

5. Conclusion

Two different numerical analyses of natural convective flow from a vertical surface heat flux with streamwise temperature differences has been assumed. We may summarize our obtained results as follows:

- For skin-friction coefficient and local Nusselt number, at the leading edge, the thickness of the fluids is larger than down edge. For large temperature wave amplitude *a*, the sinusoidal effects are greater than for small *a* in both Pr.
- The boundary-layer thickness is large for small Prandtl number.
- By comparing the numerical results of both methods in figures, we have shown that they are almost same in all regions.
- Streamlines are big for large wave amplitude *a*. The thermal-boundary layer effects are more clear and higher for large *a*.

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