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Unsteady 2D Flow in an Initially Stratified Air-filled Trapezoid

Research Article

Md. Mahafujur Rahaman[1,2], Titab Roy[3], Sidhartha Bhowmick[1*], Rabindra Nath Mondal[1] and Suvash Chandra Saha[4]

[1]*Department of Mathematics, Jagannath University, Dhaka 1100, Bangladesh*
[2]*Department of Computer Science and Engineering, Z. H. Sikder University of Science and Technology, Shariatpur 8024, Bangladesh*
[3]*Department of Mathematics, Govt. Maulana Mohammad Ali College, Tangail 1901, Bangladesh*
[4]*School of Mechanical and Mechatronic Engineering, University of Technology Sydney, Ultimo, NWS 2007, Australia*

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Abstract: Because of its widespread prevalence in both engineering sectors and nature, natural convection is extensively researched, especially in a valley-shaped trapezoidal enclosure. This study deals with a trapezoidal cavity which is initially filled with linearly stratified air. Though the side walls remain adiabatic, the temperatures of the top wall and the bottom wall are same as their adjacent fluid temperature. Natural convection in the cavity is simulated in two dimensions using numerical simulations. The Rayleigh numbers (Ra) for 10^5, 10^6, 10^7 and 10^8 have been used with a Prandtl number, Pr = 0.71 and a constant aspect ratio, A = 0.5 to find the flow characteristics. According to the numerical simulations, the development of transient flow within the trapezoid owing to the predefined conditions for the boundary may be categorized into three stages: early, transitional, and steady or unsteady. The primary flow characteristics at each of the three phases and the impact of the Rayleigh numbers on the flow's growth are stated in this study.

Keywords: *Stratified air* ● *Trapezoidal cavity* ● *Natural convection* ● *Transient flow*

1. Introduction

Natural convection in various geometries has gotten a lot of attention in recent decades because of its numerous uses, including electronic components, canned food cooling, heating and preservation, solar collectors, nuclear reactors, double-panel windows, and heat exchangers. For several engineering systems as well as geophysical scenarios where the enclosure geometry changes or incorporates additional tending walls, a triangular, square or rectangular cavity is insufficient. Because of their widespread prevalence, natural convection flows within an enclosure with additional tending walls seemed widely researched. Saha (2011) used an enhanced scaling technique and explicit numerical simulations to discover the fluid flow and heat transport within a triangular enclosure owing to sudden heating on the inclined sides. Bhowmick *et al.* (2018) explored unsteady natural convection in a valley-shaped triangular cavity filled with stratified water using two-dimensional numerical method. Again, transition to a chaotic flow was studied by Bhowmick *et al.* (2019) in a V-shaped triangular enclosure heated from below and cooled from the top.

* Corresponding Author: Sidhartha Bhowmick
Email: sidharthabhowmick@yahoo.com

Saha *et al.* (2010a) examined heat transport through attics of houses under actual thermal forcing. Again, Saha *et al.* (2010b) presented a fundamental investigation of fluid dynamics within an attic-shaped triangular enclosure with chilly upper walls and an adiabatic horizontal bottom wall.

Because of the inclined walls, studying natural convection in a trapezoidal enclosure is much more challenging than in traditional triangle, square or rectangular enclosures. In mesh generation and code development, this complicated geometry necessitates a precise and big effect. However, various investigations were accessible on natural convection focused on trapezoidal enclosures. Iyican *et al.* (1980) considered trapezoidal cavity with boundary conditions of heated bottom wall and parallel cylindrical cooled top wall to investigate natural convection of the cavity, using experimental and computational methods. Lee (1984) reported a theoretical and experimental investigation of the non-rectangular enclosure, in which two 45° inclined sides of a trapezoidal cross-section was chosen, one cooled and the other heated. Parallel and insulated were the other two sides of the enclosure. Lam *et al.* (1989) found analogous findings for a trapezoidal enclosure with an inclined cooled top wall, horizontal heated bottom wall, and vertical insulated sidewalls. Lee (1991) and Peri (1993) showed numerical findings in case of a laminar natural convection within a trapezoidal cavity with inclined sidewalls kept at varying constant temperatures and adiabatic top and bottom walls for the Rayleigh number $\leq 10^6$.

Boussaid *et al.* (2003) examined thermosolutal heat transport inside a trapezoidal chamber where the bottom was heated and the tending upper half was chilled. Mustafa and Ghani (2012) explored a natural convection flow inside a trapezoidal cavity with partially heated bottom wall and cooled vertical walls through a constant temperature bath, and a well-insulated top wall. Gholizadeh *et al.* (2016) studied the natural convection inside a trapezoidal cavity where the right inclined wall was partially heated, by means of the finite difference method. To better understand the effect of heating length on the active bottom wall, Gowda *et al.* (2019) investigated natural convection within the cavity of a trapezoid under the condition that the bottom wall was partly heated, the top wall was adiabatic, and the inclined wall were kept at a fixed cooled temperature.

Various boundary conditions were used to the trapezoidal cavity by various researchers, according to the above literature study. However, the development of natural convection flows in a valley-shaped trapezoidal cavity with linearly stratified air, is still ambiguous, which encourages this research. Using two-dimensional numerical simulations for Ra = 10^5, 10^6, 10^7 and 10^8, Pr = 0.71, and A = 0.5, the transitional flow in the trapezoidal enclosure is studied in this paper.

2. Problem formulations

This study considers a trapezoidal enclosure of height H, as well as the horizontal length of the top, $2L$, where $L = 2H$; *i.e.*, $A = H/L = 0.5$. Fig. 1 illustrates physical model with boundary conditions. A tiny percentage (= 4% of L) of each top corner is sliced to dispense with the singularity around the position between inclination and upper walls, and the cutting walls are subject to an adiabatic thermal state. The fluid in the cavity with Pr = 0.71 is considered, which is initially linearly stratified with the highest temperature of $T = T_h$ at the bottom and the lowest temperature of $T = T_c$ at the top. The boundaries are non-slip.

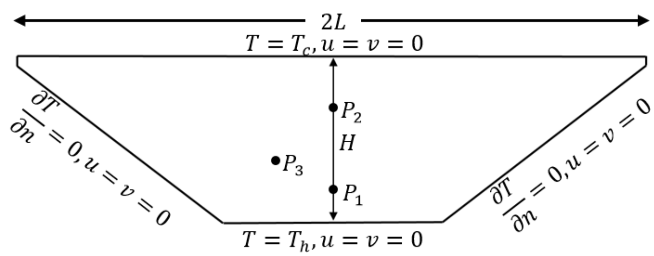

Figure 1. Schematic of physical domain with boundary conditions.

The subsequent set of governing equations with the Boussinesq approximation regulate the progress of natural convection flows in the trapezoidal enclosure:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}\right), \quad (2)$$

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right) + g\beta(T - T_0), \quad (3)$$

$$\frac{\partial T}{\partial t} + u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \kappa\left(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}\right). \quad (4)$$

Where u and v are the velocity components in the horizontal x and vertical y directions; T is the temperature; p is the pressure; g is the gravitational force and ρ, β, ν and κ are the density, thermal expansion coefficient, kinematic viscosity and thermal diffusivity of the fluid respectively.

The three parameters, which are aspect ratio (A), Prandtl number, Pr and Rayleigh number, Ra (Saha *et al.*

(2010a)), influence the natural convection and heat transmission in the cavity that are expressed as follows:

$$Ra = \frac{g\beta(T_h - T_c)H^3}{\nu\kappa}, Pr = \frac{\nu}{\kappa}, A = \frac{H}{L}. \quad (5)$$

3. Time step and grid dependent tests

In this paper, ANSYS FLUENT 17.0, a finite-volume based fluid simulation software, is used to enable the high Rayleigh number flows (Armfield and Street (1999; 2002; 2003)). In order to find solutions to the central Equations (2.1) to (2.4), and other conditions, the SIMPLE scheme is used. Using the QUICK scheme (see Leonard and Mokhtari (1990)), the advection term was discretized. Central differencing along with second order accuracy was also used to discretize the diffusion terms. Moreover, a second-order implicit time-marching scheme is employed for the unsteady term.

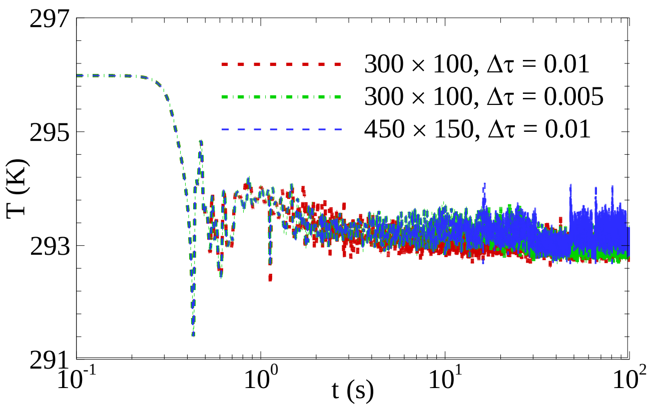

Figure 2. Temperature time series at P_1 (0, 0.03) for Ra = 10^8 with distinct grids and time steps.

In this study, the grid and time step for the highest Rayleigh number, Ra = 10^8 has been conducted based on numerical procedures. It is expected that the mesh selected for the highest Rayleigh number will also be applicable for all the lowest Rayleigh numbers. Two symmetrical meshes of 300 ×100 and 450 ×150 have been created, which are non-uniform using the application ANSYS ICEM, by way of coarser grids in the interior area as well as finer grids around the edges.

At position P_1 (0, 0.03), employing two different grids together with two different time steps, the temperature time series is computed for Ra = 10^8 as depicted in Fig. 2. The results evidently show that temperatures predicted with various meshes and time steps are constant in the initial stages, but slightly deviate in the fully developing stages. The differences in the results produced with distinct grids together with time steps is satisfactory. Considering the computational cost, a mesh of 300 × 100 and a time step of 0.01 were used in numerical simulations.

4. Results and discussions

For Ra = 10^5, 10^6, 10^7 and 10^8, Pr = 0.71 and A = 0.5, the transient development of the flow in an initially stratified air-filled trapezoidal cavity have been explained in response to consistent heating through the base and similar cooling via the top surfaces using numerical simulations.

4.1 Development of the transient flow

For Ra = 10^5, 10^6, 10^7 and 10^8, the general characteristics of flow development in a trapezoidal enclosure are presented. The development of the flow for these Rayleigh numbers, according to the numerical simulations, may be divided into the following: early stage, transitional stage, and steady or unsteady stage.

4.1.1 Flow at the early stage

In the beginning of the numerical studies, the instant conditions for isotherms are created across the surfaces that first cool the cavity via the upper surface and then make it hot through the bottom. Thermal boundary layer form along all internal surfaces as a result. The progress of the layers of the thermal boundary through time is depicted in Fig. 3, exhibiting isotherms and streamlines at various periods after start-up. Because the bottom parts of the top wall are heated and the top section is cooled, the heated fluid from the bottom travels via the boundary layer towards the upper parts. On the other hand, the boundary layer transports cooled air from the top to the bottom. Both hot and cold air meet in the middle of the top wall and release into the core. The isotherms and streamlines for different Ra remain symmetric with regard to the cavity’s x = 0 line at this stage.

4.1.2 Transitional stage

The formation of convictive instabilities marks the flow in the form of ascending and descending plumes at the transitional phase. Through the warming of the bottom portion, the horizontal thermal boundary layer, which has warmed air under colder air, is vulnerable to ‘Rayleigh-Benard instabilities’, is framed. In this regime, Fig. 4 displays the streamlines and isotherms at various periods. Subsequently, the heated air plume then moves to the cavity’s core, while the cooled air plume goes to the lower portion from the upper layer. The flow grows stronger and finally asymmetric as time passes due to pitchfork bifurcation. The following figures demonstrate the asymmetric isotherms and streamlines for the bigger Rayleigh numbers. It is without a doubt significant that with the greater Rayleigh number, the flow oscillates for quite a long period.

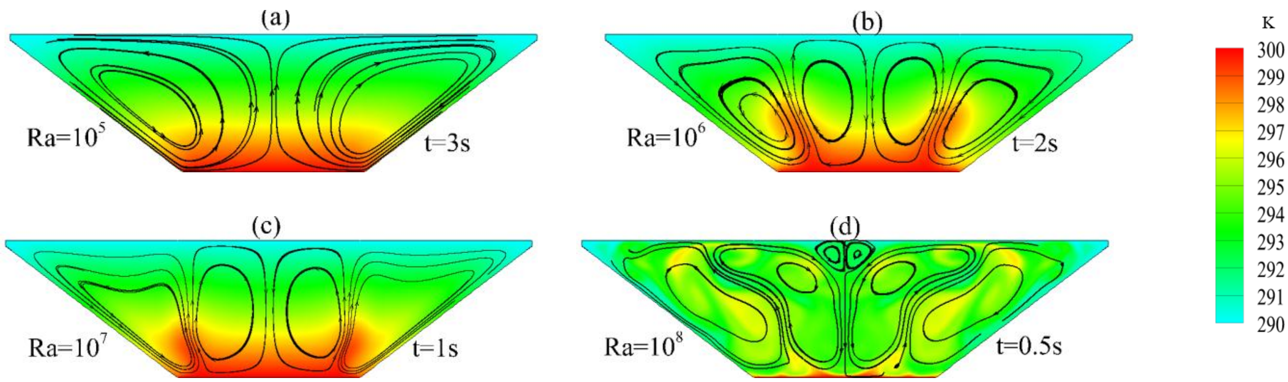

Figure 3. Streamlines and isotherms at the early stage for different Rayleigh numbers.

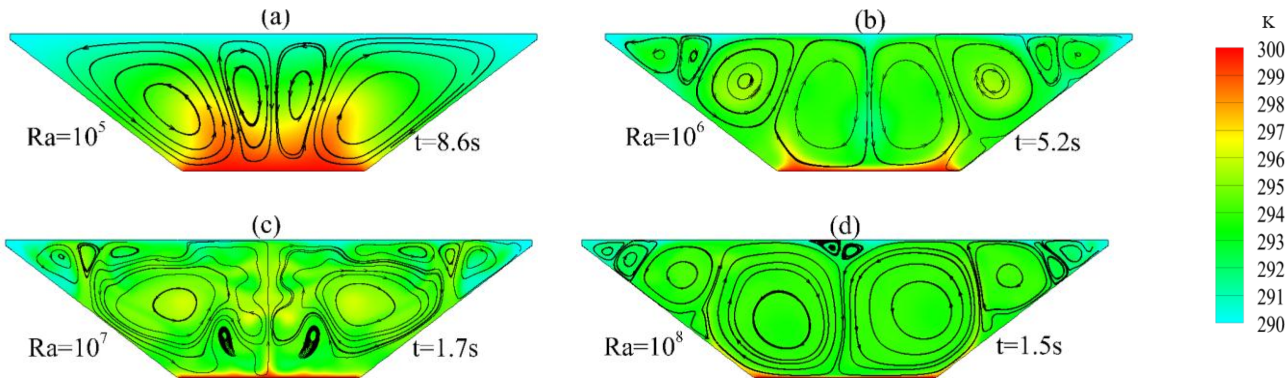

Figure 4. Streamlines and isotherms at the transitional stage for different Rayleigh numbers.

4.1.3 Flow at the steady or unsteady stage

The pitchfork bifurcation occurs early in the numerical simulation, as previously mentioned. The convective instabilities alternate on either side of the cavity, and the upward-moving heated air plumes on the base side appears in the middle at different times, which is a fascinating event as shown in the numerical simulation. During the transitional stage, the flow, on the other hand, has multiple undershoots and overshoots prior to becoming completely stable. At the fully developed stage, the fluid inside the enclosure reached a steady state for Ra = 10^5 and 10^6. Fig. 5(a) represents a few tiny cells form on the top right and left sides of the larger cell. However, when looking at the numerical simulation, it can be seen that the two tiny cells alternately emerge, indicating that the flow arrives unsteady state at a fully advanced stage for Ra = 10^7. With the increase of Ra, however, both the cells developed in the center of the two biggest cells, as seen in Fig. 5(b). In Fig. 5(b), for Ra = 10^8, the biggest two cells in the center also travels between right and left. As a result, the unsteady flow gets more complicated.

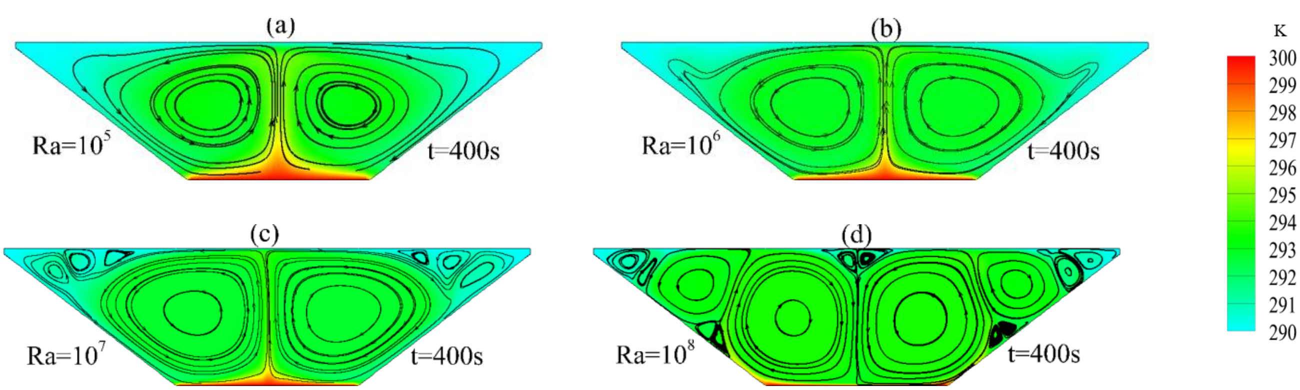

Figure 5. Streamlines and isotherms at the fully developed stage for different Rayleigh numbers.

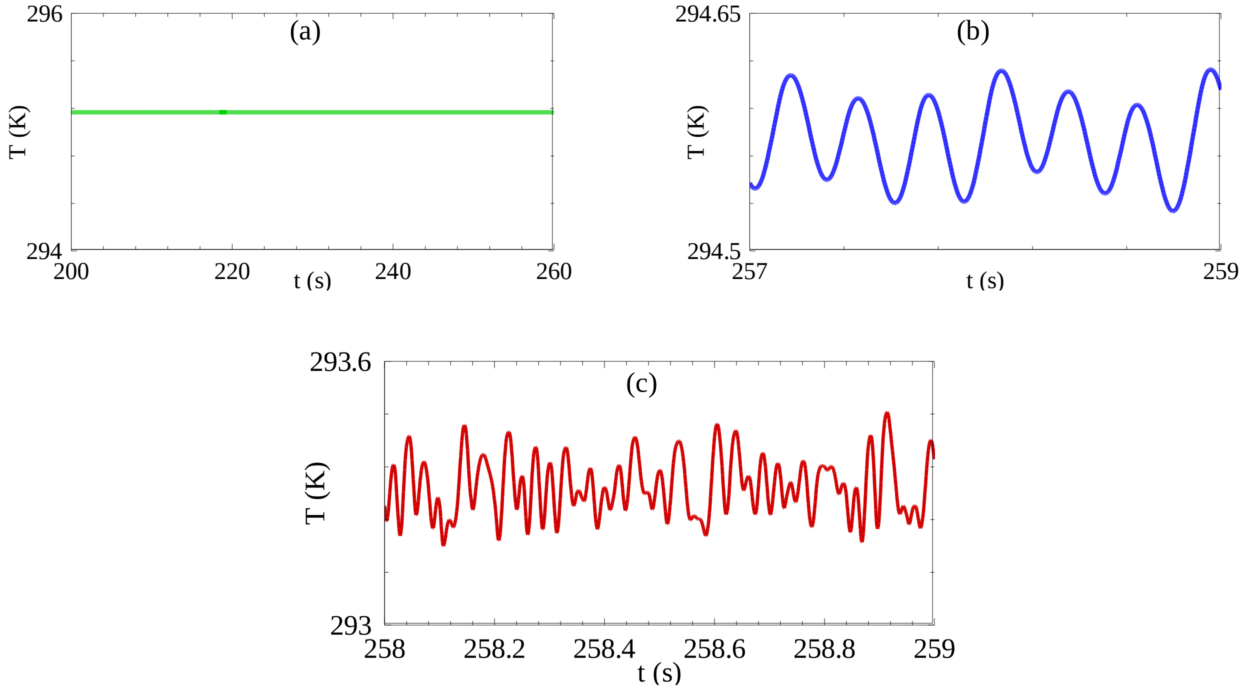

Figure 6. At the fully developed stage, temperature time series at P_3 (-0.04, 0.038) for (a) Ra = 10^6, (b) Ra = 10^7 and (c) Ra = 10^8.

To comprehend the unsteady flow for greater Rayleigh numbers, the series of temperature-time is presented in Fig. 6. The flow appears to be steady at this advanced stage for Ra = 10^6, as seen in Fig. 6 (a), and that the flow becomes periodic for Ra = 10^7 shown in Fig. 6(b). With the increase of Ra, the periodic flow fluctuates, as well as the unsteady flow turns chaotic for Ra = 10^8. This is illustrated in Fig. 6(c).

4.2 Impact of Rayleigh numbers on the progress of the flow

The Rayleigh numbers (Ra) for 10^5, 10^6, 10^7 and 10^8 have been used in the simulations. For A = 0.5, the isotherms and accompanying streamlines are depicted in Figs. 3-5 for different Rayleigh numbers. The numerical findings for the various Rayleigh numbers, as shown in Fig. 7, show some differences. To begin with, convective flow instabilities can be noticed at the lower most Rayleigh number. However, with a higher Rayleigh number, the unsteadiness become more pronounced, as well as the corresponding wave number rises. The flow becomes asymmetric and steady for Ra = 10^5 and 10^6. For Ra = 10^7 and Ra = 10^8, the flow becomes periodic and chaotic, respectively.

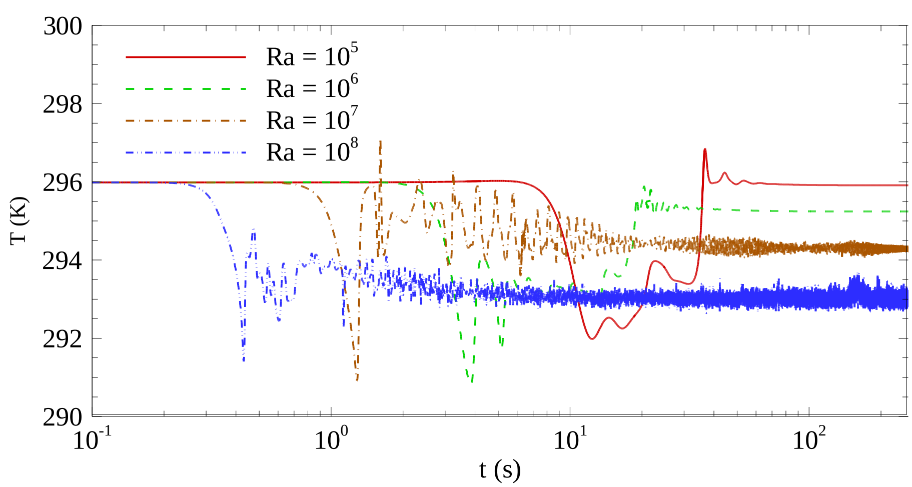

Figure 7. Series of temperature and time at point P_2 (0, 0.06) for various Rayleigh numbers.

5. Conclusions

The present study deals with the transient thermal convection in trapezoidal enclosure which is a stratified air-filled. Here, the enclosure's top wall is cooled at a certain lower temperature, while bottom is heated with a fixed aspect ratio A = 0.5 throughout different Rayleigh numbers (Ra = 10^5, 10^6, 10^7 and 10^8). According to numerical simulation, the development of transient flow within the enclosure owing to the predefined conditions for boundary may be categorized into three separate stages: early, transitional, and steady or unsteady, all of which are shown in Fig. 3 to Fig. 6. The flow, at the beginning phase, is portrayed through the arrangement of thermal boundary layers close towards every internal surface and the commencement of primary circulations. In the energy equations, whenever the terms of convection and conduction are adjusted, the flow gets into the transitional state. In the transitional stage, the flow is depicted via the base of convective instabilities through ascending and descending thermal plumes, as well as the creation of the cellular flow formations. Furthermore, symmetric flows regarding the geometrically symmetric plane for smaller Ra, as well as for relatively higher Ra is characterized by pitchfork bifurcation which represents the flow from symmetry to asymmetry.

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