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# Phase Portrait and Stability Analysis of Two Species Competition Models

Research Article

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**ABSTRACT**

The competition raises significant environmental concerns among species that share habitats. The density of a population's own population, as well as the density of populations with which it is in competition, determines the rate at which that population changes. This study employs a competition model predicated on the hypothesis that the two groups have an antagonistic effect on one another's demographic trends. It was based on the idea that two populations of a given species share a habitat but compete with one another for limited resources. Both the abrogation equilibrium and the coexistence equilibrium are found to be unstable. Relationships between species, finding equilibrium points, and species habits are broken down into their fundamental components. Different competition model states and their solutions are investigated. A weak competition that is asymptotically stable on a global scale is presented at some equilibrium points. In the end, the phase picture is plotted, and the various stability models used for the competition are expressed.

**Keywords:** *Equilibrium, Species, Phase portrait, Jacobian matrix, Stability, Eigen value*

## 1. Introduction

In mathematical biology, competition modeling is incriminating spectacles. It is outstretched in nature as two species compete for the same limited provision one of the species fall outs elided by Murry (2007). The modeling is ticklish to perform in any generality for population dynamics. An ecosystem is tenacious to encircle that presume its interactions to defection whereas a chance of severe controls. Competition model was enacting introduced as an ethos of cross-linguistic saying procedure by Whinney *et al. (1989).* Gotelli and Levin (1967) brainchild in conceptual and discourse ecology, the competition species coexistence has been a center motive for confined provisions. By general migration rates and competition exorbitance, it imparts a complete analytical portrayal of the competition upshot. Undertake that the completion is well-proportioned and the ambience is homogeneous by Nguyen Huu Khanh. Windarto Windarto and Eridani Eridani argued that competition inferiors the suitability of both organisms inlaid, since the availability of one of the ingredient incessantly assuage the content of provision incessantly procurable to isolate. In many habitats and ecosystems, diverse sort of interactions interim embodied souls attained. There are two kinds of interactions such as intraspecific and

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interactions in ecological interaction. Interactions interim isolates of the analogous species are proposed intraspecific and interactions interim two or more species are proposed interspecific interactions. Competition, predation, herbariums and symbiosis include interspecific interactions. The weaker species has a negative impact in ecological community by Lang *et al*. (2013). Booker *et al.* (2009) an accumulation of populations imparts directly and indirectly of at least two diverse species within a specific geographical area. For survival isolate can contend for food, water, space, light, mates, or any other provision. *Begon et al. (2014) deliberated* for competition to occur the provision must be confined and the population swells exponentially as the species may attain a lavish extent of each provision. Direct intraspcific competition also includes claimed animals in an area that may not later have actual conflicts with other species and is at risk of conflict with the dominant member. Organisms can accomplish indirectly and only notify indirectly through engaged resources. Apparent competition occurs between the former populations. *Norbury et al* (2001) speculates that an enhancement in the population of predatory species, such as exploitative competition, will get more predators in the region, increasing a person’s craving and abating their caliber to survive. Ni *et al.* (2018) depict Lyapunov or upper–lower solutions guarantee global stability of nonnegative constant equilibria. For significant nonlocal intra-specific competition coefficients, the nonlocal diffusive competition model may have nonconstant positive equilibria for weak competition. Gumu (2014) studied discrete-time dynamic model equilibrium point local and global stability conditions with and without the Allee effect. Hassel *et al.* (1976) present a similar two-age class model, in which intra- and inter-specific competition coexist in one of the developmental stages. The Classical Lotka-Volterra competition equation revealed by Gavina *et al.* (2018) that competing species can only survive if intra-specific rivalry outweighs the inter-specific competition. As shown by Yu *et al.* (2020), nonlinear harvesting can display more complicated dynamic behavior than linear harvesting. Averill *et al*. (2017) investigated two-person mathematical model species with the same population dynamics in space and time, regionally, but has remained static throughout the years. One species goes extinct, or the two species attain a state of coexistence equilibrium in the long run, as demonstrated by Chen *et al*. (2022). It is shown that the traditional Lotka-Volterra competition model can be transformed into a cooperative system Alhasanat *et al.* (2019). In this study, Ma *et al*. (2019) examine the effects of bistable nonlinearity on the Lotka-Volterra competition model by looking at the speed and direction of moving waves. Using a Lotka-Volterra diffusive competitive system in a sphere with two unbounded in a spherically symmetric setting, Du *et al.* (2018) examine the dispersal invading species.

Ecological familiarities are typically classified by exorcism of their interspecific interactions.

**Table 1:** Denomination of interspecific familiarity.

| Interaction type | Competing for a determinate food supply for the similar species | Effect on species 1 | Effect on species 2 |
|---|---|---|---|
| **Competition Model** | Food supply is abundant the number of individuals | Positive(+) | Positive(+) |
| | The amount of this food supply available to each individuals | Negative(-) | Negative(-) |

Table 1 narrates this denomination, most momentous of which is competition. Interaction type consequence on Species 1 consequence on Species 2 Competition (+) positive (+) positive and (–) Negative (–) Negative.

## 2. Formulation of Competition Model

Two and higher dimensional systems are used in differential equations for some of the problems of population dynamics which starts with a population of two species occupies the same residence. Competes for limited resources and it is to get some insight into environmental issues. The logistic equation for a single species: $\frac{dN}{dt} = rN(1-\frac{N}{K})$ (1)

Where $r$ is the growth (birth) rate, and $K$ the carrying capacity. $N_1(t)$ and $N_2(t)$ to illustrate the populations of the two species at time $t$. The Competition model for diminution growth rate of species, say $N_1$, concerning of species $N_2$ is to replace the growth rate part $(1-\frac{N}{K})$ in the logistic equation by the subsequent forms:

$$\left.\begin{aligned}\frac{dN_1}{dt} &= N_1(r_1 - p_1 N_1 - q_1 N_2) \\ \frac{dN_2}{dt} &= N_2(r_2 - p_2 N_1 - q_2 N_2)\end{aligned}\right\} \quad (2)$$

Here, $r_i$ is the maximal per capita growth of species $i$, $p_1, p_2, q_1, q_2$ are the parameters for the first and second species gradually. All the parameters are obtained to be positive.

## 3. Obtaining Equilibrium Points

For equilibrium points by determining the values of $N_1$ and $N_2$ such that $\frac{dN_1}{dt} = 0$ and $\frac{dN_2}{dt} = 0$ is verified. By determining values for which

$$\frac{dN_1}{dt} = 0. \quad (3)$$

$N_1 = 0$ Or $r_1 - p_1 N_1 - q_1 N_2 = 0$

$$r_1 - p_1 N_1 - q_1 N_2 = 0$$

$$\Rightarrow p_1 N_1 = r_1 - q_1 N_2$$

$$\Rightarrow N_1 = \frac{r_1 - q_1 N_2}{P_1} \quad (4)$$

By determining values for which $\frac{dN_2}{dt} = 0$

$$N_2(r_2 - p_2 N_1 - q_2 N_2) = 0$$

$$N_2 = 0 \text{ or } r_2 - p_2 N_1 - q_2 N_2 = 0 \quad (5)$$

$$\Rightarrow q_2 N_2 = r_2 - p_2 N_1$$

$$\therefore N_2 = \frac{r_2 - p_2 N_1}{q_2} \quad (6)$$

Setting $N_1 = 0$ in the equation (5), it is seen that

$$r_2 - p_2.0 - q_2 N_2 = 0$$

$$\Rightarrow r_2 - 0 - q_2 N_2 = 0$$

$$\Rightarrow N_2 = \frac{r_2}{q_2}$$

Setting $N_2 = 0$ in the equation (4) it seems that

$$N_1 = \frac{r_1}{p_1}$$

To ascertain in consequence of equilibrium points,

$$E_1(N_1, N_2) = (0, 0) \quad (7)$$

$$E_2(N_1, N_2) = (\frac{r_1}{p_1}, 0) \quad (8)$$

$$E_3(N_1, N_2) = (0, \frac{r_2}{p_2}) \quad (9)$$

$$E_4(N_1, N_2) = (\frac{r_1 - q_1 N_2}{p_1}, \frac{r_2 - p_2 N_1}{q_2}) \quad (10)$$

The equilibrium point in equation (10) may not be positive for all feasible values.

**Table 2:** Behavior of Competition model, described by equation (2) for different equilibrium points.

| Equilibrium Points | $N_1$ | $N_2$ | Behavior of Species |
|---|---|---|---|
| $E_1(N_1,N_2)=(0,0)$ | 0 | 0 | Both species extinct. |
| $E_2(N_1,N_2)=(\frac{r_1}{p_1},0)$ | $\frac{r_1}{p_1}$ | 0 | $N_1$ Prevails, $N_2$ goes extinct. |
| $E_3(N_1,N_2)=(0,\frac{r_2}{p_2})$ | 0 | $\frac{r_2}{p_2}$ | $N_1$ goes extinct , $N_2$ Prevails. |
| $E_4(N_1,N_2)=(\frac{r_1-q_1N_2}{p_1},\frac{r_2-p_2N_1}{q_2})$ | $\frac{r_1-q_1N_2}{p_1}$ | $\frac{r_2-p_2N_1}{q_2}$ | Both species co-exist. |

## 4. Local Stability Analysis

It is analyzed the stability of the model (2) each equilibrium solution separately based of the corresponding Jacobian matrix may be approximated by

$$J(N_1,N_2)=\begin{bmatrix} r_1-2p_1N_1-q_1N_2 & -q_1N_1 \\ -q_2N_2 & r_2-2p_2N_2-q_2N_2 \end{bmatrix} \tag{11}$$

**Proposition 1:**

***The equilibrium state*** $E_1, (p_1,p_2,q_1,q_2,r_1,r_2)>0$ ***of the competition model (2) is unstable as*** $\lambda_1=r_1$, *and* $\lambda_2=r_2$.

**Proof:** From model (2), $E_1(N_1,N_2)=(0,0)$

To recapitulate the stability setting $N_1=0+w_1(t)$,

$$N_2=0+w_2(t) \tag{12}$$

From the equation (2) and (12)

$$\Rightarrow \left.\begin{aligned} \frac{dN_1}{dt} &= w_1(r_1-p_1w_1-q_1w_2) \\ \frac{dN_2}{dt} &= w_2(r_2-p_2w_1-q_2w_2) \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} \frac{dN_1}{dt} &= w_1r_1-p_1w_1{}^2-q_1w_2 \\ \frac{dN_2}{dt} &= w_2r_2-w_1w_2-q_2w_2{}^2 \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} \frac{dN_1}{dt} &\approx r_1w_1 \\ \frac{dN_2}{dt} &\approx r_2w_2 \end{aligned}\right\} \tag{13}$$

In matrix form this can be written as

$$\begin{pmatrix} \frac{dN_1}{dt} \\ \frac{dN_2}{dt} \end{pmatrix} \approx \begin{pmatrix} r_1 & 0 \\ 0 & r_2 \end{pmatrix}\begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \tag{14}$$

The characteristic matrix is

$$\begin{pmatrix} r_1 & 0 \\ 0 & r_2 \end{pmatrix} - \lambda\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The characteristic equation is $\begin{vmatrix} r_1-\lambda & 0 \\ 0 & r_2-\lambda \end{vmatrix}=0$

$$\Rightarrow (r_1-\lambda)(r_2-\lambda)=0$$

Either $(r_1-\lambda)=0$, Or $(r_2-\lambda)=0$

$$\Rightarrow \lambda=r_1, \text{ Or } \lambda=r_2$$

Here, $p_1,p_2,q_1,q_2,r_1,r_2>0$

$$\Rightarrow \lambda_1=r_1, \text{ Or } \lambda_2=r_2$$

The equilibrium state $E_1$ of the competition model (2) is unstable as it has two positive values $\lambda_1 = r_1$, and other $\lambda_2 = r_2$.

**Proposition 2:** ***The equilibrium state*** $E_2$, $(p_1, p_2, q_1, q_2, r_1, r_2) > 0$ ***of the competition model (2) is unstable if*** $p_1 r_2 > q_2 r_1$ ***or stable if*** $p_1 r_2 < q_2 r_1$ ***as*** $\lambda_1 = -r_1$, *and* $\lambda_2 = r_2 - \frac{p_2 r_1}{p_1}$.

**Proof:** From model (2), $E_2(N_1, N_2) = (\frac{r_1}{p_1}, 0)$ To recapitulate the stability setting $N_1 = \frac{r_1}{p_1} + w_1(t)$,

$$N_2 = 0 + w_2(t) \tag{15}$$

From the equation (2) and (15)

$$\left.\begin{aligned} \frac{dN_1}{dt} &= \left(\frac{r_1}{p_1} + w_1\right)\left(r_1 - \left(p_1\left(\frac{r_1}{p_1} + w_1\right) - q_2 w_2\right)\right) \\ \frac{dN_2}{dt} &= w_2\left(r_2 - \left(\frac{r_1}{p_1}\right) - q_2 w_2\right) \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} \frac{dN_1}{dt} &= \left(\frac{r_1}{p_1} + w_1\right)\left(r_1 - r_1 - p_1 w_1 - q_2 w_2\right) \\ \frac{dN_2}{dt} &= w_2\left(r_2 - \frac{p_2 r_1}{p_1} - p_2 w_1 - q_2 w_2\right) \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} \frac{dN_1}{dt} &= \left(\frac{r_1}{p_1} + w_1\right)\left(-p_1 w_1 - q_2 w_2\right) \\ \frac{dN_2}{dt} &= w_2\left(r_2 - \frac{p_2 r_1}{p_1} - p_2 w_1 - q_2 w_2\right) \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} \frac{dN_1}{dt} &\approx -r_1 w_1 - \frac{r_1 q_2}{p_1} w_2 \\ \frac{dN_2}{dt} &\approx w_2\, r_2 - \frac{p_2 r_1}{p_1} w_2 \end{aligned}\right\} \tag{16}$$

In matrix form this can be written as

$$\begin{pmatrix} \frac{dN_1}{dt} \\ \frac{dN_2}{dt} \end{pmatrix} \approx \begin{pmatrix} -r_1 & \frac{r_1 q_1}{p_1} \\ 0 & r_2 - \frac{p_2 r_1}{p_1} \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix} \tag{17}$$

The characteristic matrix is

$$\begin{pmatrix} -r_1 & \frac{r_1 q_1}{p_1} \\ 0 & r2 - \frac{p_2 r_1}{p_1} \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The characteristic equation is

$$\begin{vmatrix} -r_1 - \lambda & -\frac{r_2 q_2}{p_1} \\ 0 & r_2 - \frac{p_2 r_1}{p_1} - \lambda \end{vmatrix} = 0$$

The characteristic matrix is

$$\Rightarrow \left(-r_1 - \lambda\right)\left(r_2 - \frac{p_2 r_1}{p_1} - \lambda\right) = 0$$

Either $\left(-r_1 - \lambda\right) = 0$, Or $\left(r_2 - \frac{p_2 r_1}{p_1} - \lambda\right) = 0$

$$\Rightarrow \lambda = -r_1, \text{ Or } \lambda = r_2 - \frac{p_2 r_1}{p_1}$$

Here, $p_1, p_2, q_1, q_2, r_1, r_2 > 0$

$$\Rightarrow \lambda_1 = -r_1, \text{ Or } \lambda_2 = r_2 - \frac{p_2 r_1}{p_1}$$

The equilibrium state $E_2$ of the competition model (2) is unstable or stable as it has one negative value $\lambda_1 = -r_1$, and other $\lambda_2 = r_2 - \frac{p_2 r_1}{p_1}$.

**Proposition 3:** ***The equilibrium state*** $E_3$, $(p_1, p_2, q_1, q_2, r_1, r_2) > 0$ ***of the competition model (2) is unstable if*** $p_1 r_2 > q_2 r_1$ ***or stable if*** $p_1 r_2 < q_2 r_1$ ***as*** $\lambda_1 = -r_2$,

*and* $\lambda_2 = \frac{p_2 r_1 - q_1 r_2}{p_2}$.

**Proof:** From model (2), $E_2(N_1, N_2) = (0, \frac{r_2}{p_2})$

To recapitulate the stability setting $N_1 = 0 + w_1(t)$,

$$N_2 = \frac{r_2}{p_2} + w_2(t) \tag{18}$$

From the equation (2) and (18)

$$\left.\begin{aligned} \frac{dN_1}{dt} &= w_1\left(r_1 - (p_1 w_1) - q_1\left(\frac{r_2}{p_2} w_2\right)\right) \\ \frac{dN_2}{dt} &= \left(\frac{r_2}{p_2} + w_2\right)\left(r_2 - p_2 w_1 - q_2\left(\frac{r_2}{p_2} + w_2\right)\right) \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} \frac{dN_1}{dt} &= w_1 r_1 - p_1 w_1{}^2 - \frac{q_1 r_2}{p_2} w_1 - q_1 w_1 w_2 \\ \frac{dN_2}{dt} &= w_2\left(r_2 - p_2 w_1 - \frac{q_2 r_2}{p2} - q_2 w_2\right) \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} \frac{dN_1}{dt} &= w_1 r_1 - p_1 w_1{}^2 - \frac{q_1 r_2}{p_2} w_1 - q_1 w_1 w_2 \\ \frac{dN_2}{dt} &= \frac{r_2{}^2}{p_2} - r_2 p_2 w_1 - \frac{q_2 r_2}{p_2{}^2} - \frac{r_2}{p_2} q_2 w_2 + w_2 r_2 - p_2 w_1 w_2 - \frac{q_2 r_2}{p_2} w_2 - w_2{}^2 q_2 \end{aligned}\right\}$$

$$\Rightarrow \left.\begin{aligned} \frac{dN_1}{dt} &\approx -r_1 w_1 - \frac{r_2 q_1}{p_2} w_1 \\ \frac{dN_2}{dt} &\approx -p_2 r_2 w_1 - \frac{q_2 r_2}{p_2} w_2 + w_2 r_2 - \frac{q_2 r_2}{p_2} w_2 \end{aligned}\right\}$$

In matrix form this can be written as

$$\begin{pmatrix} \frac{dN_1}{dt} \\ \frac{dN_2}{dt} \end{pmatrix} \approx \begin{pmatrix} -r_1 - \frac{q_1 r_2}{p_2} & 0 \\ -r_2 p_2 & -\frac{r_2}{p_2} q_2 + r_2 - \frac{q_2 r_2}{p_2} \end{pmatrix} \begin{pmatrix} w_1 \\ w_2 \end{pmatrix}$$

The characteristic matrix is

$$\begin{pmatrix} -r_1 - \frac{q_1 r_2}{p_2} & 0 \\ -r_2 p_2 & -\frac{r_2}{p_2} q_2 + r_2 - \frac{q_2 r_2}{p_2} \end{pmatrix} - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

The characteristic equation is

$$\begin{vmatrix} -r_1 - \frac{q_1 r_2}{p_2} - \lambda & 0 \\ 0 & r_2 - \left(\frac{q_2 r_1}{p_2} - 1 + \frac{q_2}{p_2}\right) - \lambda \end{vmatrix} = 0$$

$$\Rightarrow \left(-r_1 - \frac{q_1 r_2}{p_2} - \lambda\right)\left(-\frac{r_2 q_2}{p_2} + r_2 - \frac{q_2 r_2}{p_2} - \lambda\right) = 0$$

Either $\left(-r_1 - \frac{q_1 r_2}{p_2} - \lambda\right) = 0$,

Or $\left(-\frac{q_2}{p_2} + r_2 - \frac{q_2 r_1}{p_2} - \lambda\right) = 0$

$\Rightarrow \lambda = -r_2$, Or $\lambda = r_1 - \frac{q_1 r_2}{p_2}$

Here, $p_1, p_2, q_1, q_2, r_1, r_2 > 0$

$\Rightarrow \lambda_1 = -r_2$, Or $\lambda_2 = \frac{p_2 r_1 - q_1 r_2}{p_2}$

The equilibrium state $E_2$ of the competition model (2) is unstable or stable as it has one negative value $\lambda_1 = -r_2$, and other $\lambda_2 = \frac{p_2 r_1 - q_1 r_2}{p_2}$

## 5. Numerical Illustrations

### 5.1 Results and Discussions

**Competition Model 1:**

The following model that describes the competition model between two species

$$\left.\begin{aligned} \frac{dN_1}{dt} &= N_1(17 - 6N_1 - 2N_2) \\ \frac{dN_2}{dt} &= N_2(15 - 2N_1 - N_2) \end{aligned}\right\} \tag{19}$$

A coupled set of ordinary differential equations is required to depict the model (i). Let us refer to the population size of the first species as $N_1(t)$ and the population size of the second species as $N_2(t)$ at any time $t(\geq 0)$. Let us assume that the first species has an intrinsic per capita growth rate of 17 and the second species has an intrinsic per capita growth rate of 15. Absolute intra-specific competition coefficients are 6 and 2. The inter-specific competition coefficients are absolute values of 2 and 1, respectively. It is expected that all of the parameters have positive values. Solving the equation (19) the critical points occur at $E_1 = (0,0)$, $E_2 = \left(\frac{17}{6},0\right)$, $E_3 = (0,15)$, $E_4 = \left(-\frac{13}{2},28\right)$.

### 5.1.1 Plotting the phase portrait for the given system of differential equation (19)

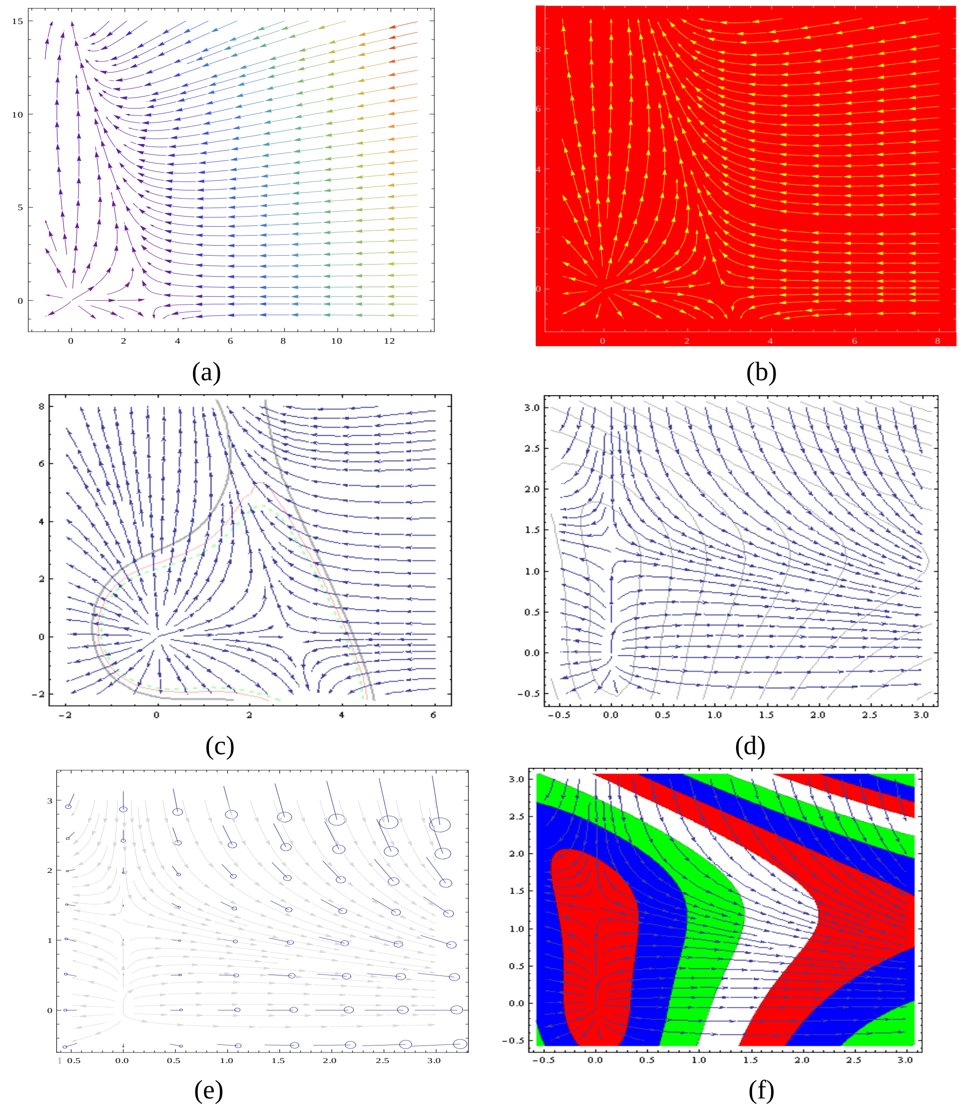


**Fig. 1:** Phase portraits for the competition model (1) by using equation (19). Boundary trajectories are not shown. All interior trajectories a periodic and enclose the interior steady states: (a), (b) at (0, 0), (c) at (0, 0), (2.5, 0), (d) (0, 0), (1.5, 0), and (e), (f) at (0, 0), (1.5, 0). Linear stability for real eigenvalues $(i)\,\lambda_1, \lambda_2 < 0$ (stable node), $(ii)\,\lambda_1, \lambda_2 > 0$ (unstable node).

The Jacobian matrix for the system of equation (19) is

$$J(N_1, N_2) = \begin{bmatrix} 17-12N_1-2N_2 & -2N_1 \\ -2N_2 & 15-2N_1-2N_2 \end{bmatrix}$$

At $E_1 = (0,0)$

$$J(E_1) = J(N_1, N_2) = \begin{bmatrix} 17 & 0 \\ 0 & 15 \end{bmatrix}$$

At $E_2 = \left(\frac{17}{6}, 0\right)$

$$J(E_2) = J(N_1, N_2) = \begin{bmatrix} 34 & -\frac{17}{3} \\ 0 & \frac{28}{3} \end{bmatrix}$$

At $E_3 = (0,15)$

$$J(E_3) = J(N_1, N_2) = \begin{bmatrix} -13 & 0 \\ -30 & -15 \end{bmatrix}$$

At $E_4 = \left(-\frac{13}{2}, 28\right)$

$$J(E_4) = J(N_1, N_2) = \begin{bmatrix} 39 & 13 \\ -56 & -28 \end{bmatrix}$$

Since the eigenvalue of $J(E_1) = 17$ and 15, the point (origin) is unstable node because the linearized matrix has two positive eigenvalues. The eigenvalue $J(E_2) = \frac{28}{3}$ and 0, the point $E_2$ cannot analyze it is not isolated (one eigen value is zero). The eigen value of $J(E_3) = -13$ and 0, the point $E_3$ is asymptotically stable because their jacobian matrix has 0 eigen value and negative eigen value. The eigen value of $J(E_4) = 2i\sqrt{182}$ and $-2i\sqrt{182}$ is unstable because the jacobian matrix has one positive eigen value and one negative eigenvalue.

**Competition Model 2 :**

$$\left.\begin{aligned} \frac{dN_1}{dt} &= N_1(24 - 7N_1 - 2N_2) \\ \frac{dN_2}{dt} &= N_2(21 - 5N_1 - N_2) \end{aligned}\right\} \quad (20)$$

One requires a system of coupled ordinary differential equations to depict the model (ii) properly. It is used the abbreviations $N_1(t)$ and $N_2(t)$ to denote the populations of the first and second species, respectively $t(\geq 0)$. Consider two species, one of which has an intrinsic growth rate per person of 24 and the other of which has an intrinsic growth rate per person of 21. In absolute terms, the competition coefficients within species are 7 and 5, respectively. In comparison to inter-specific competition, the intra-specific competition coefficient is two, and the latter is 1. All of the parameters are expected to have positive values. Solving the equation (20) the critical points occur at $E_1 = (0,0)$, $E_2 = (0, 21)$, $E_3 = \left(\frac{24}{7}, 0\right)$, $E_4 = (6, -9)$.

### 5.1.2 Plotting the phase portrait for the given system of differential equation (20)

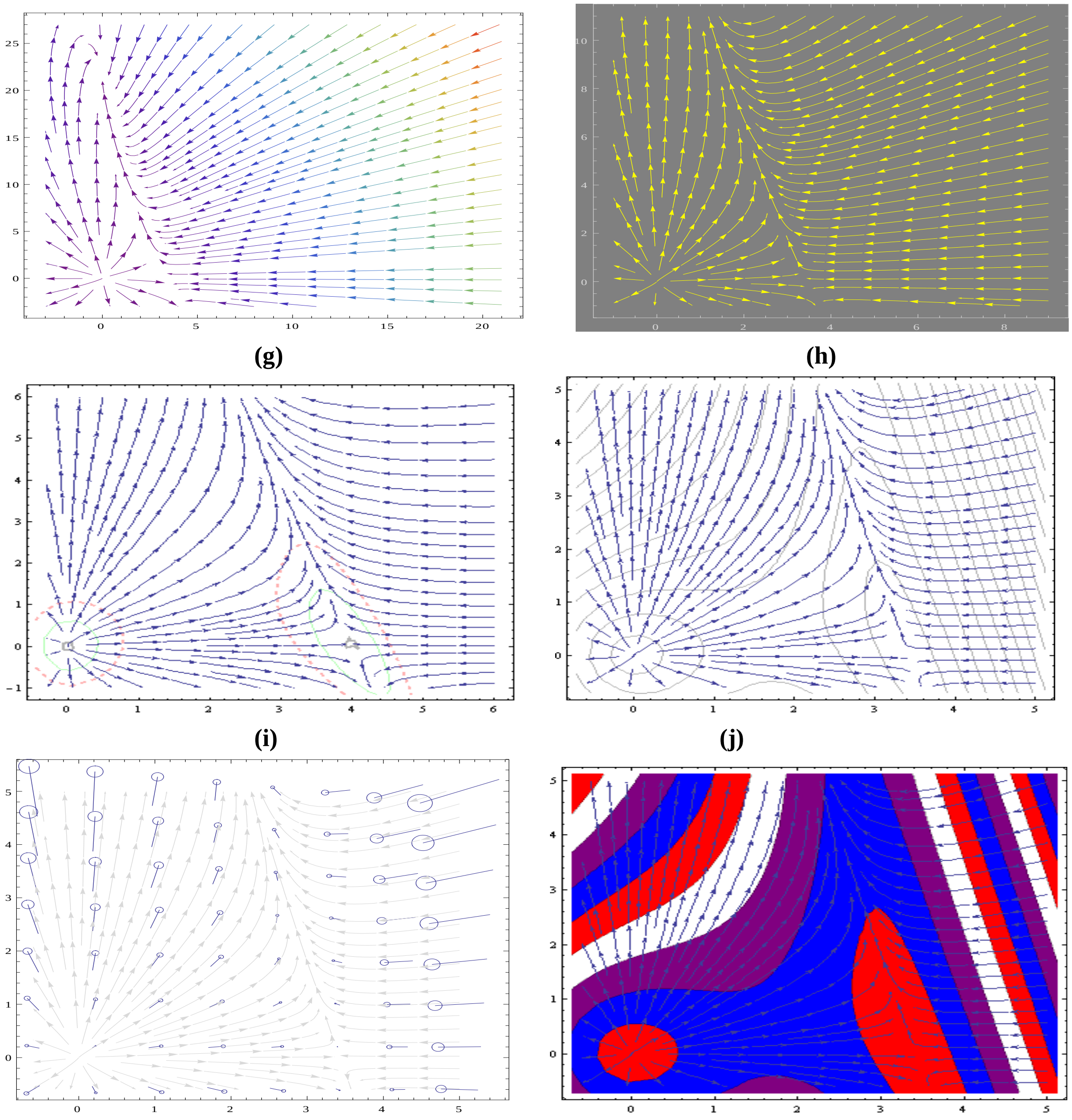



**Fig. 2:** Phase portraits for the competition model (2) by using equation (20). Boundary trajectories are not shown. All interior trajectories a periodic and enclose the interior steady states: (g) at (0, 22.5), (h) (0, 0), (i) at (0, 0), (4, 0), (j) (0, 0), (3.5, 0), and (k), (l) at (0, 0), (3.5, 0). The interior steady state now exists and is locally stable, asymptomatically stable and unstable with a spiral.

The Jacobian matrix for the system of equation (20) is

$$J(N_1, N_2) = \begin{bmatrix} 24-14N_1-2N_2 & -2N_1 \\ -5N_2 & 21-5N_1-2N_2 \end{bmatrix}$$

At $E_1 = (0,0)$

$$J(E_1) = J(N_1, N_2) = \begin{bmatrix} 24 & 0 \\ 0 & 21 \end{bmatrix}$$

At $E_2 = (0, 21)$

$$J(E_2) = J(N_1, N_2) = \begin{bmatrix} -18 & 0 \\ -105 & -21 \end{bmatrix}$$

At $E_3 = \left(\frac{24}{7}, 0\right)$

$$J(E_3) = J(N_1, N_2) = \begin{bmatrix} -24 & -\frac{48}{7} \\ -30 & \frac{27}{7} \end{bmatrix}$$

At $E_4 = (6, -9)$

$$J(E_4) = J(N_1, N_2) = \begin{bmatrix} -42 & -12 \\ 45 & 9 \end{bmatrix}$$

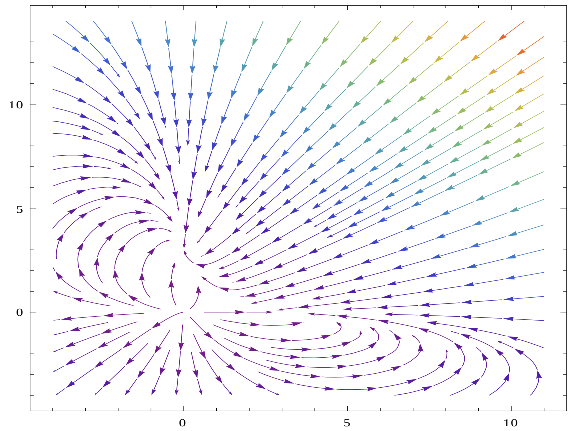
Since the eigenvalue of $J(E_1) = 24$ and 21, the point (origin) is unstable node because the linearized matrix has two positive eigenvalues. The eigenvalue $J(E_2) = -21$ and -18, the point $E_2$ is asymptotically stable as both eigen value is negative. The eigen value of $J(E_3) = -24$ and $\frac{27}{7}$ the point $E_3$ is unstable because $E_3$ is a saddle point or their jacobian matrix has one negative eigen value and one positive eigen value. The eigen value of $J(E_4) = -27$ and $-6$ is asymptotically stable because both eigen value is negative.

**Competition Model 3:**

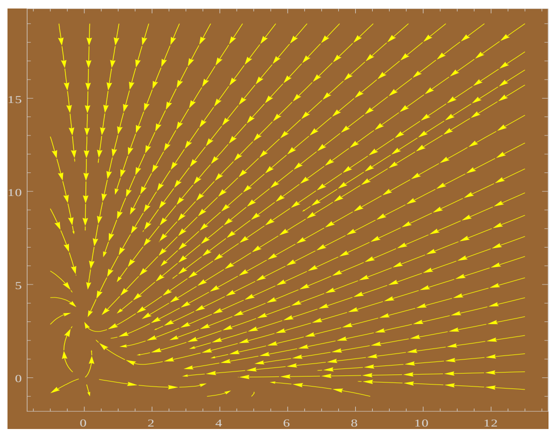
$$\left.\begin{aligned} \frac{dN_1}{dt} &= N_1(6 - 2N_1 - 5N_2) \\ \frac{dN_2}{dt} &= N_2(9 - 4N_1 - 3N_2) \end{aligned}\right\} \qquad (21)$$

To represent the model, a coupled set of ordinary differential equations is needed. At any moment $t(\geq 0)$, the population of the first species will be denoted by $N_1(t)$ , while the population of the second species will be denoted by $N_2(t)$. Assume that the intrinsic per capita growth rate for the first species is 6 and that for the second species it is 9. The coefficients of absolute intra-specific competition are 2 and 4, respectively. The inter-specific competition coefficients are absolute values of 5 and 3 respectively. Each parameter's value should be positive.

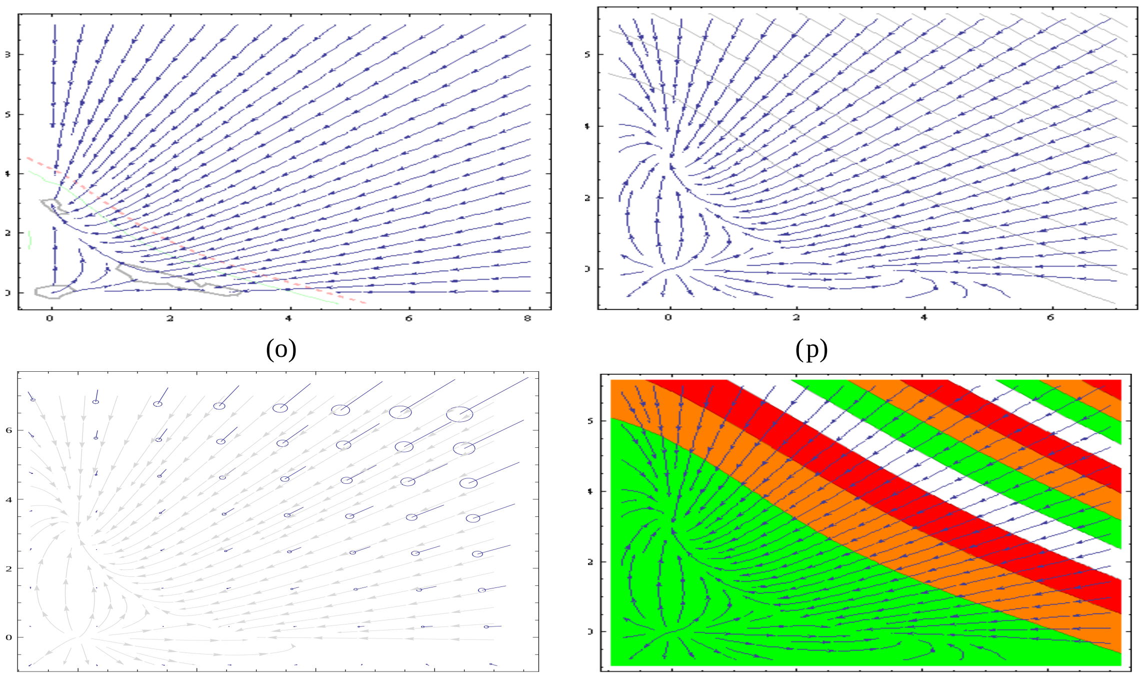
**5.1.3 Plotting the phase portrait for the given system of differential equation (21)**




(m)




(n)



(o) (p)

(q) (r)

**Fig. 3:** Phase portraits for the competition mode (3) by using equation (21). Boundary trajectories are not shown. All interior trajectories a periodic and enclose the unique interior steady states: (m) at (0, 0), (0, 2.5), (n) (0, 5), (o), (p), (q) and (r) at (0, 0), (0, 2.5). The interior steady state now exists and is locally stable and unstable with a spiral.

The Jacobian matrix for the system of equation (21) is

$$J(N_1, N_2) = \begin{bmatrix} 6-4N_1-5N_2 & -5N_1 \\ -4N_2 & 9-4N_1-6N_2 \end{bmatrix}$$

At $E_1 = (0,0)$

$$J(E_1) = J(N_1, N_2) = \begin{bmatrix} 6 & 0 \\ 0 & 9 \end{bmatrix}$$

At $E_2 = (3,0)$

$$J(E_2) = J(N_1, N_2) = \begin{bmatrix} -6 & -15 \\ 0 & -3 \end{bmatrix}$$

At $E_3 = (0,3)$

$$J(E_3) = J(N_1, N_2) = \begin{bmatrix} -9 & 0 \\ -12 & -9 \end{bmatrix}$$

At $E_4 = \left(\frac{17}{24}, \frac{3}{7}\right)$

$$J(E_4) = J(N_1, N_2) = \begin{bmatrix} \frac{43}{42} & -\frac{85}{24} \\ -\frac{12}{7} & \frac{151}{42} \end{bmatrix}$$

Since the eigenvalue of $J(E_1) = 9$ and 6, the point (origin) is unstable node because the linearized matrix has two positive eigenvalues. The eigen value of $J(E_2) = -3$ and 0, the point $E_2$ is asymptotically stable because their jacobian matrix has 0 eigen value and negative eigen value. The eigen value of $J(E_3) = -9$ and 0, the point $E_3$ is asymptotically stable because their jacobian matrix has 0 eigen value and negative eigen value. The eigen value of $J(E_4) = \frac{1}{168}(615 + \sqrt{213385})$ and $\frac{1}{168}(615 - \sqrt{213385})$ is unstable because the jacobian matrix has one positive eigen value and one negative eigenvalue.

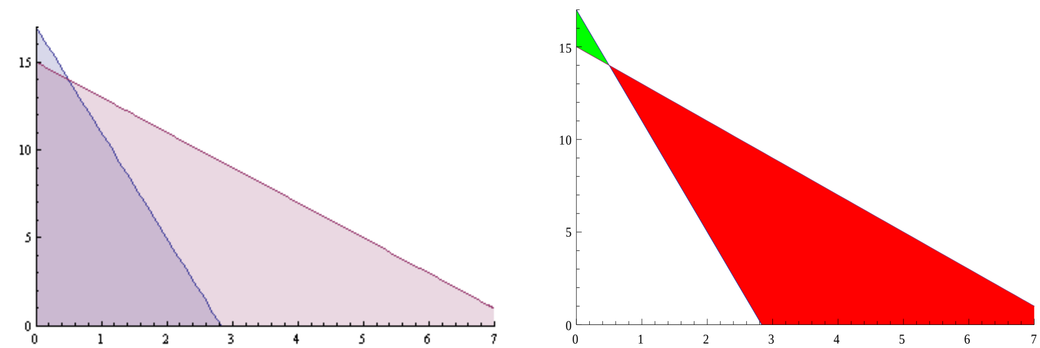



**Fig. 4:** Plotting domains related to competition of model (1) by using the equation (19). The null clines for the competition model 1. $\frac{dN_1}{dt}=0$ is $N_1=0$ and $17-6N_1-2N_2=0$ with $\frac{dN_2}{dt}=0$ being $N_2=0$ and $15-2N_1-N_2=0$.

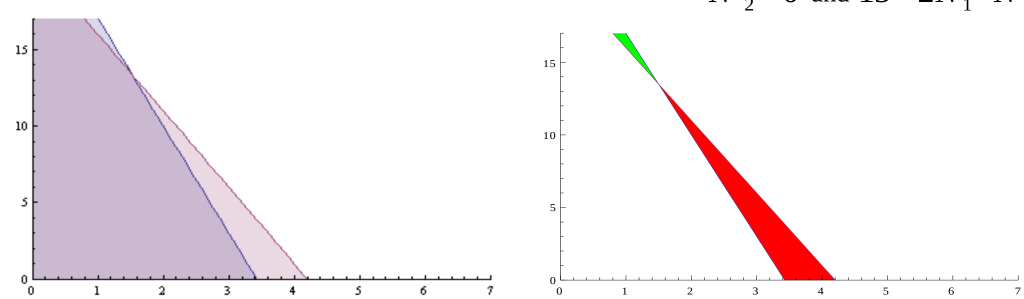



**Fig. 5:** Plotting domains related to competition of model (2) by using the equation (20). The null clines for the competition model 1. $\frac{dN_1}{dt}=0$ is $N_1=0$ and $24-7N_1-2N_2=0$ with $\frac{dN_2}{dt}=0$ being $N_2=0$ and $21-5N_1-N_2=0$.

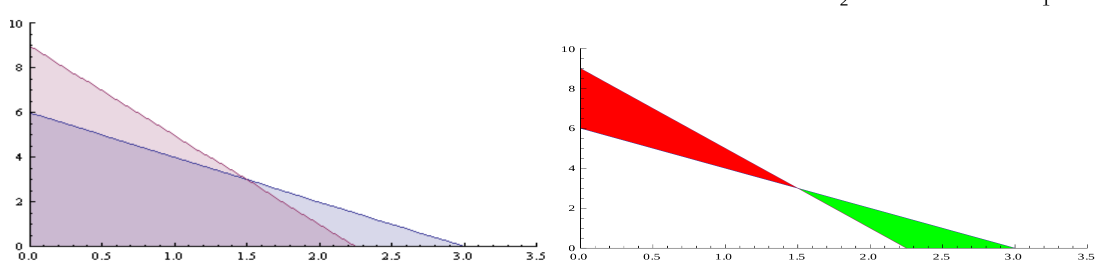



**Fig. 6:** Plotting domains related to competition of model (3) by using the equation (21).

The null clines for the competition model 3. $\frac{dN_1}{dt}=0$ is $N_1=0$ and $6-2N_1-5N_2=0$ with $\frac{dN_2}{dt}=0$ being $N_2=0$ and $9-4N_1-3N_2=0$.

If two lines don not confine through the first quadrilateral, then the two species cannot coexist amicably and at least one of them will die. This policy is called Gauss's competitive exclusion policy.

## 6. Global Stability Analysis

Positive equilibrium of the equation number (2) is globally asymptotic stable for weak competition case as $\frac{q_2}{p_1}<\frac{r_2}{r_1}<\frac{p_2}{q_1}$. See shi (2018).

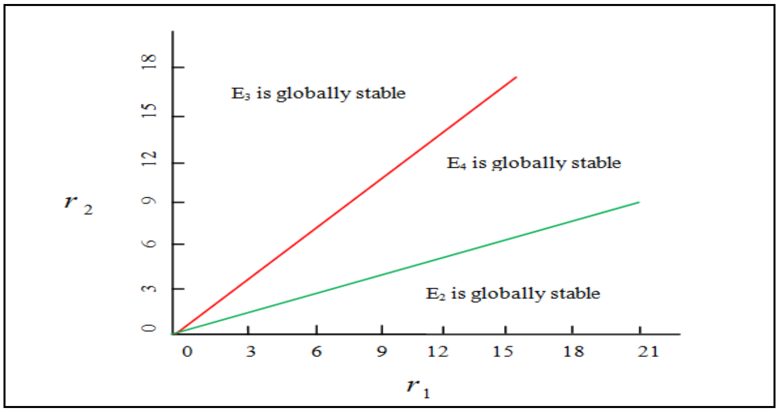



**Fig. 7:** Global stability analysis of the equation (19). Taking $r_1 = 17, r_2 = 15, p_1 = 6, q_1 = 2, q_2 = 1, p_2 = 2$.

Here, $\frac{p_2}{q_1} = \frac{2}{2} = 1$, $\frac{r_2}{r_1} = \frac{15}{17} = 0.88$, $\frac{q_2}{p_1} = \frac{1}{6} = 0.17$

(a) $\frac{r_2}{r_1} < \frac{p_2}{q_1}$, $E_2$ is globally asymptotically stable;

(b) $\frac{r_2}{r_1} > \frac{q_2}{p_1}$, $E_3$ is globally asymptotically stable;

(c) $\frac{q_2}{p_1} < \frac{r_2}{r_1} < \frac{p_2}{q_1}$., $E_4$ is globally asymptotically stable;

## 7. Conclusion

The competition model is a repeating experiment with two species. Differential equation-based models assist us in comprehending the current condition of the ecological population model. The need to find solutions to these problems has become more urgent as it has been clear that natural inversions are a global concern that contributes to climatic fluctuation. The dynamic character of the provision is sought at equilibrium points, and the dynamic nature of the system close to the solution trades stability for a loss of steadiness due to the varying parameters. There are several attainable equilibriums in this model, including the abrogation equilibrium, the abrogation of the first equilibrium, the abrogation of the second equilibrium, and the coexistence population equilibrium. The extinction equilibrium and the coexistence equilibrium have both been observed to be unstable. In addition to this, it was discovered that the extinction of the original population is conditionally asymptotically stable. Weak competition and an illustration of globally asymptotically stable equilibrium exist at some equilibrium points. The intersection of the nuclleolines, the degree to which they adapt, their relative position, and the stability evaluation of each equilibrium point by the flow of trajectories in the phase plane are used to determine the nature of the vector field.

## References


Averill I**,** Lam KY and Lou Y. (2017). “The Role of Advection in a Two-Species Competition Model: A Bifurcation Approach”, *Memoirs of the American Mathematical Society*, 245: 1161.

Alhasanat A, & Ou C. (2019). “Minimal-Speed Selection of Traveling Waves to the Lotka–Volterra Competition Model”, Journal of Differential Equations , 266 : 7357-7378.

Booker RW, Callaway, R.M, Cavieres, LA, Kikvidze, Z., Lortie, C.J., Michalet, R. & Pugnaire, F.I., Banuet, A. V., & Whitham, G.(2009). Don't Diss Integration: A Comment on Ricklefs's Disintegrating Communities. American Naturalist 174: 919-927.

. Begon M, Howarth, RW, & Townsend C. (2014) R. Townsend, Essentials of Ecology, 4th edition: 103-105.

Chen S, Shi J, Shuai Z, & Wu, Y. (2022). "Global Dynamics of a Lotka–Volterra Competition Patch Model", *London Mathematical Society*, Nonlinearity, 35: 817-842.

Du Y, & Wu CH. (2018). "Spreading with Two Speeds and Mass Segregation in a Diffusive Competition System with Free Boundaries", *Calculus of Variations and Partial Differential Equations.* Springer, 57: 52.

Gotelli NJ, A Primer of Ecology, 4th edition, Sinauer Associates, Inc., Sunder- land, MA.

Gumu OA. (2014). "Global and Local Stability Analysis in a Nonlinear Discrete-time Population Model" Springer, *Journal of Advances in Differential Equations*, 249.

Gavina MK, Tahara AT, Tainaka, K H. Ito, Morita, S., Morita, G., Ichinose, T., Okabe, T., Togashi, T. Nagatani & Yoshimura, J. (2018). "Multi-species Coexistence in LotkaVolterra Competitive Systems with Crowding Effects", *Scientific Reports*, 8: 1198.

Hassel MP, & Comins HN. (1976). "Discrete Time Models for Two-Species Competition", Elsevier, *Journal of Theoretical Biology*, 9: 202-221.

Khanh NH. (2012). "Stability analysis in competition population model." 6: 11-19.

Levin S. A. (1992). "The problem of pattern and scale in ecology", Ecology, 73: 1943-1967.

Lang J. M, & Benbow, ME. (2013). "Species Interactions and Competition", *Nature Education Knowledge*, 4**:** 8.

Murray J. D., Mathematical Biology I. An Introduction, 3rd Edition, 2007.

Ma M., Huang, Z. & Ou, C. (2019). "Speed of the Traveling Wave for the Bistable Lotka-Volterra Competition Model", Nonlinearity, 32: 3143-3162.

Norbury & Grant (2001). "Conserving dryland lizards by reducing predator-mediated apparent competition and direct competition with introduced rabbits", 38: 350-1361.

Shi W. Ni, J. & Wang, M. (2018). "Global Stability and Pattern Formation in a Nonlocal Diffusive Lotka-Volterra Competition Model" *Elsevier*, 262: 6891-6932.

Whinney M, Brian *& Bates* E. (1989). "The Crosslinguistic Study of Sentence Processing", New York: *Cambridge University Press*, 20: 471-489.

Windarto W. & Eridani E. (2020). "On Modification and Application of Lotka-Volterra Competition Model." 2268: 050006.

Yu X, Zhu, Z. & Li, Z. (2020). "Stability and Bifurcation Analysis of Two-Species Competitive Model with Michaelis–Menten Type Harvesting in the First Species", Springer, *Journal of Advances in Differences Equations*, 397: 3662.