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The Study of the Chaotic Conducts of a Single Controlled Parameterized Logistic Map

Research Article

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ABSTRACT

This study analyzes the behavior of crucial features carried over from the conventional one-dimensional logistic map. The plain one-dimensional logistic map has numerous significant aspects that have been preserved in the new dimension, and the behavior of those elements is being investigated in this research. Differential equations analysis can benefit from the utilization of maps as a tool. One-dimensional differential equations are used as an example. However, maps can exhibit far more complex behaviors. It is seen that the points of trajectories that are near to each other diverge with time. The trajectories that never settle down to fixed points or periodic orbits. The trajectory of evolution can be drastically altered by even small changes in the initial conditions. While chaotic systems may appear nonlinear and random at first glance, they are in fact governed by underlying patterns. The main focus of this work is on how to locate the logistic map, as well as how to explore the chaotic behavior of the logistic equation by adjusting the governing parameters, and how to ultimately uncover bifurcation diagrams, etc.

Keywords: *Bifurcation, Control parameter, Trajectory, Logistic map, Chaos*

1. Introduction

The theory of chaos, logistic map dynamics, bifurcation theory, the self-assembly and self-organization processes, and the concept of the edge of chaos all make extensive use of dynamical systems as essential components. The study of dynamical systems is the primary emphasis of dynamical systems theory. This theory can be used to a broad number of subjects, including mathematics, physics, biology, chemistry, engineering, economics, history, and medicine, amongst others. The theory of dynamical systems is primarily concerned with the investigation of dynamical systems by Trachette et al. (2015). It explains how to change the motion of a chaotic attractor into a time-periodic motion that attracts, using only modest, time-dependent changes to a control parameter. The time-periodic motion arises because an unstable periodic orbit within the attractor has been stabilized by Filipe et al. (1992). Verhulst (1845) was the one who first put out the idea of using a logistic differential equation in his model of population expansion. According to this concept, the birthrate is the same as the product of the current population and the available resources (Kwasnicki, 2013; Girdzijauskas et al., 2012). Models of economic growth frequently employ this differential equation. It is believed that the logistic map can be seen as a discrete analog to this differential equation. The simple quadratic map, that is, the logistic map, displays universal and

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chaotic dynamics (May, 1976; Baumol et al., 1989). Focus on one-dimensional maps to introduce modeling, simulation, and visualization of discrete nonlinear dynamical systems and chaos by Geoff (2018). Applications of chaos control in physical systems, biological systems, synchronizing chaos, intimating with turmoil, and targeting trajectories of nonlinear dynamical systems by Daniel et al. (2003). From the SI fractional-order epidemic model, a discrete SI model is proposed. Existence, stability, and bifurcation are analyzed. The model flips, and Neimark–Sacker bifurcates using the center manifold theorem and bifurcation theory by Mahmoud et al. (2018). A significant shift in how they behaved was a result of a relatively little adjustment to one of the parameters. For the entire spectrum of control parameters, both the complete bifurcation diagram and the basin of attraction for the logistic map are shown by Holmgren (1994). On chaos, carry the bifurcation diagram for the logistic map for the parameter range of Frank et al. (2005), so it is easy to take it for granted when discussing it. Nevertheless, whenever one discusses the diagram, it is essential to point out that the first systematic examination of the bifurcation diagram was completed. One of the primary purposes of utilizing mathematical modeling in ecology is for the purpose of controlling and managing the dynamics of the population. One of the primary purposes of utilizing mathematical modeling in ecology is for the purpose of controlling and managing the dynamics of the population (Murray, 2002; Lewis et al., 1993; May, 1974; Sharov, et al., 1998). Habitat management, such as the development of corridors or stepping-stone population patches and the importation of additional individuals to support the target population, are also possible solutions (May et al. 1975; May et al., 1976; Zoltan et al. 2011). The initial value simulations of the full non-smooth delay differential algebraic equation are compared with the smoothed bifurcation diagrams. These simulations provide a detailed explanation of how chaotic chattering dynamics originate from the non-smooth bifurcations of periodic orbits, (Mohammad et al., 2016; Vasily et al., 2021) which mostly lend credence to the smoothing technique.

2. Methodology

Assume the logistic map

$$x_{n+1} = r x_n (1 - x_n) \tag{1}$$

Where n is the number of iteration. Simple non-linear dynamical equations can lead to intricate and chaotic behavior, as shown by the logistic map, a model of population dynamics. An analog of the continuous-time logistic map for population growth, where x_n is a dimensionless measure of the population in the n^{th} generation, and $r \geq 0$ is the intrinsic growth rate. Apply a method of local bifurcation analysis, such as period-doubling bifurcation, to the logistic map. Logistic map dynamics shifted in response to variations in the governing parameter. Many researchers assumed that accurate predictions could be made with more resources, such as better algorithms, more data, or more well-known mechanisms. The availability of resources like food, water, shelter, predators, etc. set a natural ceiling on population size. If we think about the exponentially growing limiting value model of the logistic differential equation. However, mathematicians and physicists have recently come to realize that this is not feasible. The problem is caused by chaos, a mathematical phenomenon. Greater computing power and accuracy will never produce complete predictability when the disorder is incorporated into a mathematical model. The ratio of the current population to the maximum population is represented by the number, which ranges from 0 to 1. This nonlinear difference equation must account for reproduction at low population densities and population growth at a rate proportional to the existing population. There will be famine (density-dependent mortality), where the growth rate is proportional to the number obtained by subtracting the current population from the assumed carrying capacity of the environment. Assuming the current generation's species population can be accurately estimated, it will be seen that a prediction of the population at a time in the future, t can be made using only data from the current generation. Chaos can occur in a discrete one-dimensional system. For example, $x = 0$ is constant, and populations disappear if $r > 1$, conversely, populations increase, and $x = 0$ is unstable if $r > 1$. Mathematical software like MATHEMATICA was used to analyse the chaotic results, and the resulting visualizations were used to present the findings of our study.

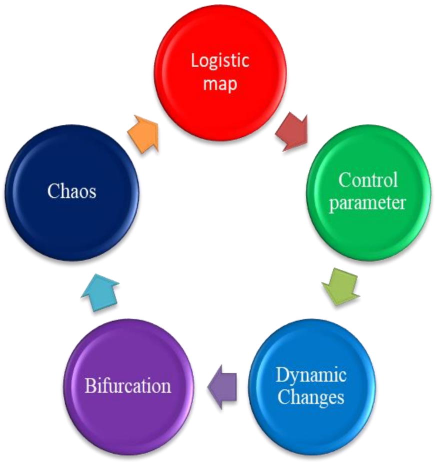

Fig. 1: Flow Chart of the logistic Map.

3. Results and Discussion

3.1 Dynamics of logistic map for control parameter values $(0 < r < 2)$ (Eric, 2008).

The orbits of neighboring seeds behave substantially differently after a few iterations because they are sensitive to the original conditions. The mathematical equation is frequently extremely challenging to solve precisely. Computers are most frequently used by scientists to obtain approximations of mathematical map answers. Unfortunately, scientists have frequently been unable to make predictions based on the results of the computer, despite significant advancements in computational speed and accuracy.

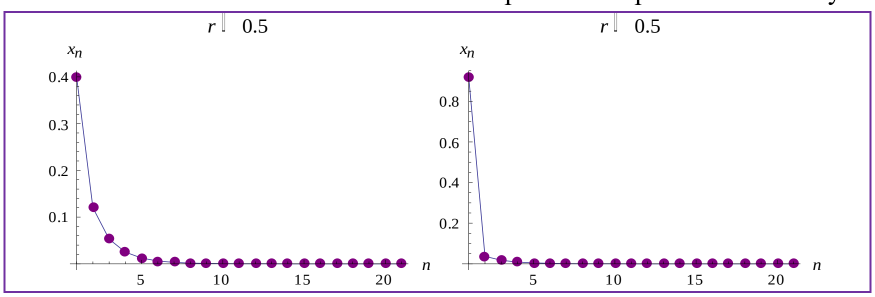

Fig. 2: Dynamics of the logistic map (1) for iteration number, $n = 20$ and $r = 0.5$.

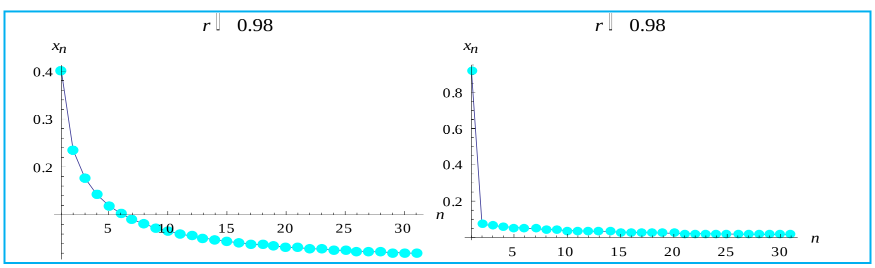

Fig. 3: Dynamics of the logistic map (1) for iteration number $n = 30$ and $r = 0.98$.

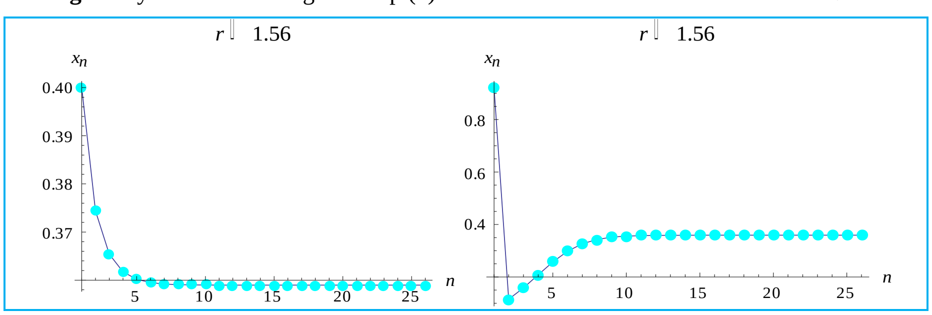

Fig. 4: Dynamics of the logistic map (1) for iteration number $n = 25$ and $r = 1.56$.

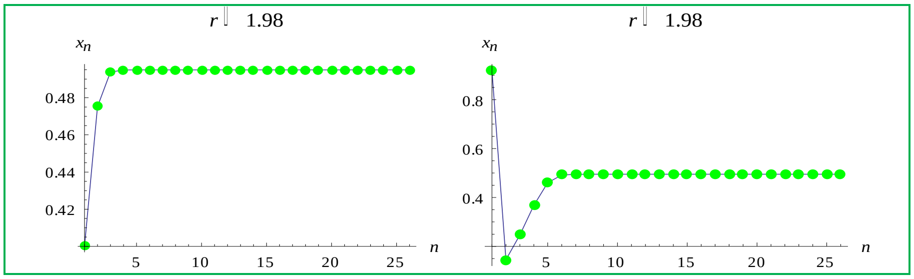

Fig. 5: Dynamics of the logistic map (1) for iteration number $n = 25$ and $r = 1.98$.

Iterate is the simple verb for continuously repeating a process. The process of iterating involves repeatedly evaluating a function. Figure 2 and figure 3 show the first 20 and 30 iterations of the map (1) are illustrated above for parameter value $r = 0.5$ and r = 0.98 Left side of Figure 2 and figure 3 behave parabolic manner. For r = 1.56, the left side diagram in figure 4 is parabolic, but the right side diagram is not parabolic. At the 25th iteration, for r = 1.98, the left and right side diagrams in figure 5 exhibit different behavior and one point in right side diagram moves downward.

3.2 Dynamics of logistic map for negative control parameter values (Eric, 2008).

Because of its accessibility, the logistic map is often employed to kick off a discussion of chaos. Chaotic systems are extremely sensitive to their initial conditions, which is one feature of the logistic map for the negative values of r that provides an elementary breakdown of anarchism.

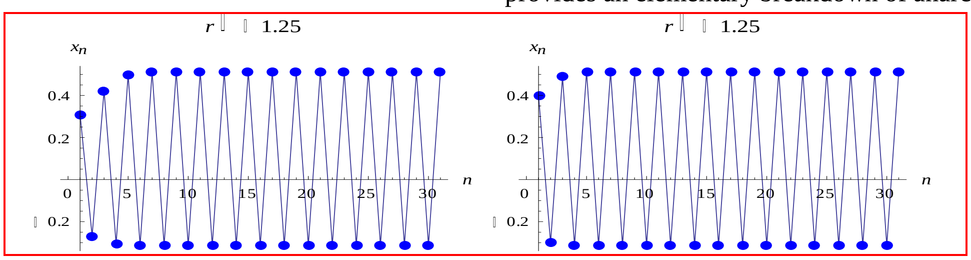

Fig. 6: Dynamic of the map (1) for iteration number $n = 30$ and $r = -1.25$.

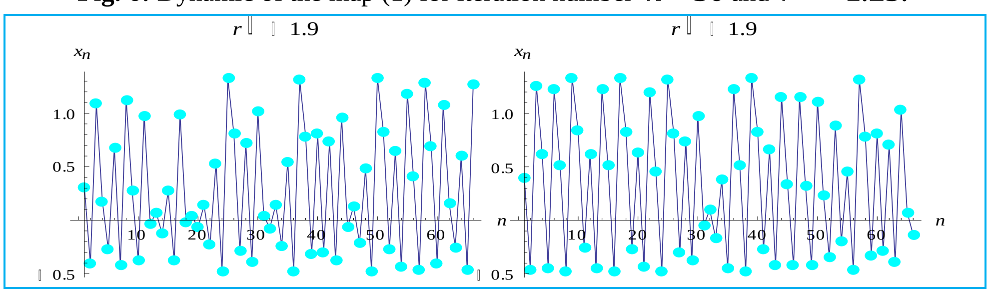

Fig. 7: Chaotic behavior of map (1) iteration number, $n = 60$ and $r = -1.9$.

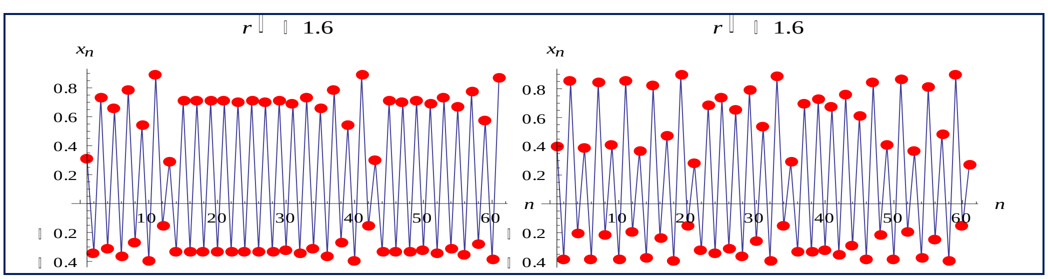

Fig. 8: Chaotic behavior of the map (1) for iteration number, $n = 65$ and $r = -1.6$.

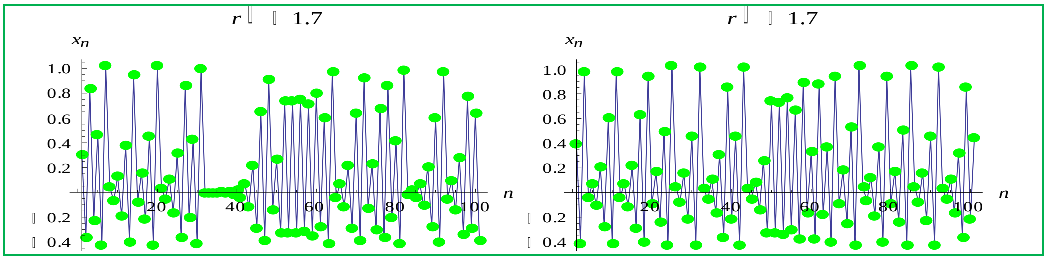

Fig. 9: Chaotic behavior of the map (1) for positive integer $n = 100$ and $r = -1.7$.

Figure 6 depicts the periodic oscillation that occurred during the first 30 iterations, along with the parameter value $r = -1.25$ of the map (1). Figures 7, 8, and 9 illustrate non-periodic oscillations for the first 60, 65 and 100 iterations of the map, respectively, for $r = -1.9, -1.6, -1.7$ values of map (1). In passing, we'd like to point out that Figures 7, 8, and 9 depict an unusual condition that leads to chaos. The statistics presented above shift both upward and downward depending on the starting value of x that is used.

3.3 Dynamics of the logistic map for control parameter values $(2 < r < 4)$ (Eric, 2008).

Oscillations begin to emerge when r more than three; with the period doubling as it increases at certain values of r, it is no longer cyclical because the doubling rate rises as the number approaches 3.57.

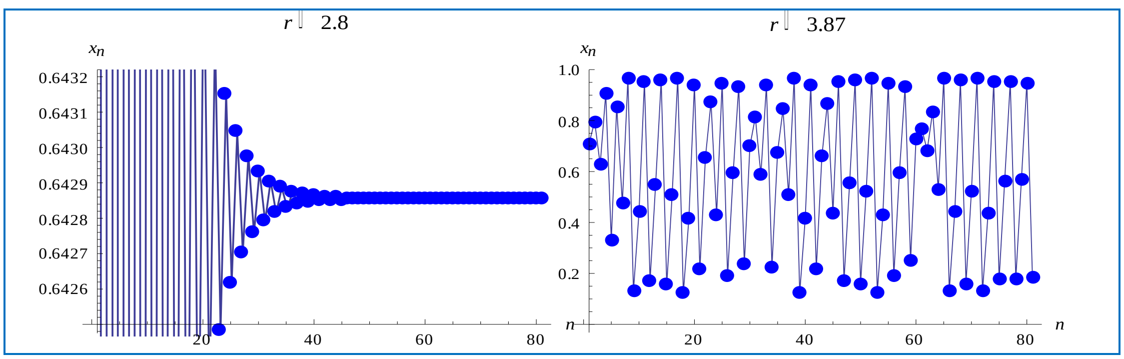

Fig. 10: Chaotic behavior of the logistic map (1) for iteration number, $n = 80$ and $r = 2.8, 3.87$.

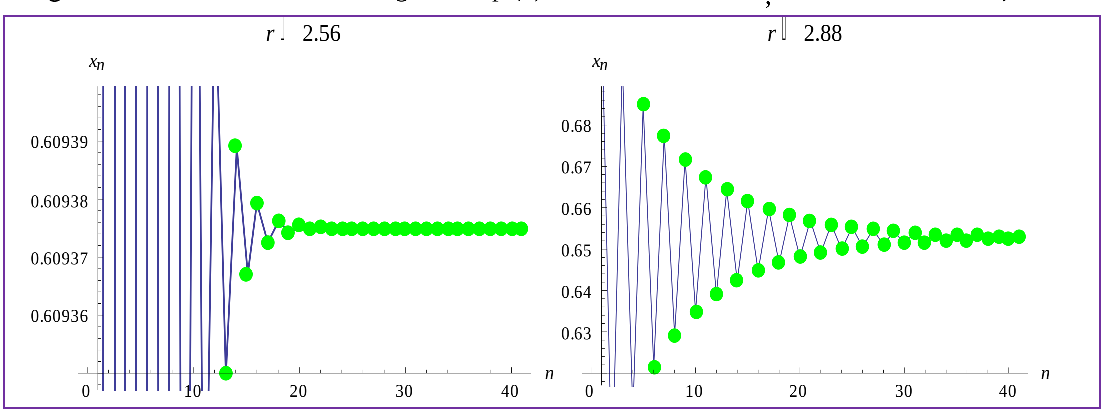

Fig. 11: Chaotic behavior of the logistic map (1) for iteration number, $n = 40$ and $r = 2.56, 2.88$.

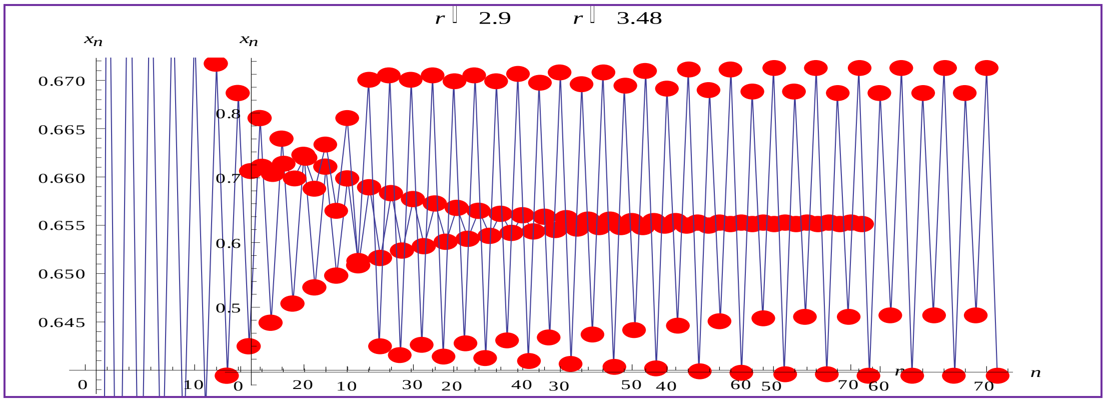

Fig. 12: Chaotic behavior of the map (1) for iteration number, $n = 70$ and $r = 2.9,\ 3.48$.

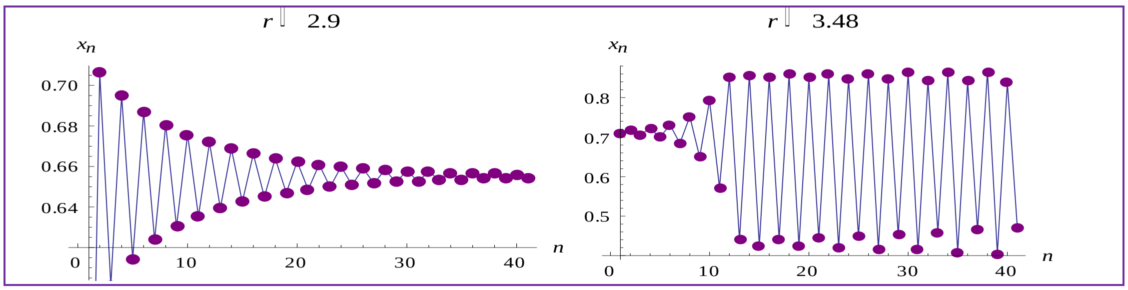

Fig. 13: Chaotic behavior of the map (1) for iteration number, $n = 40$ and $r = 2.96, 3.48$.

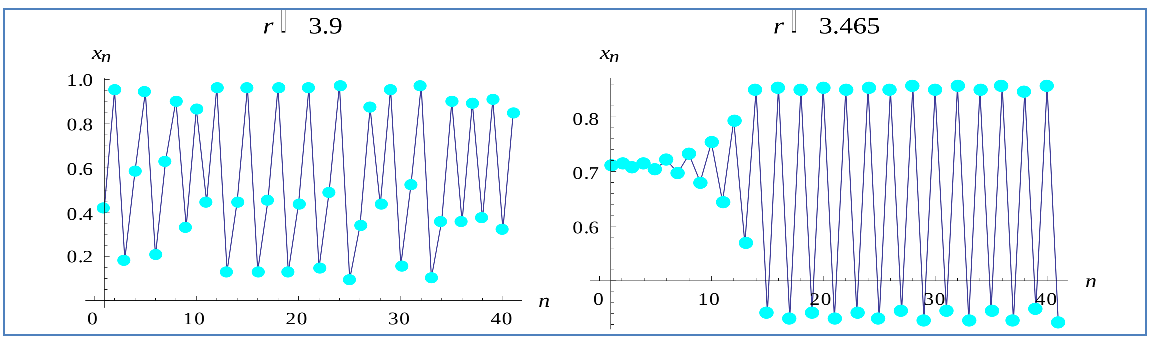

Fig. 14: Chaotic behavior of the map (1) for iteration number, $n = 40$ and $r = 3.9, 3.465$.

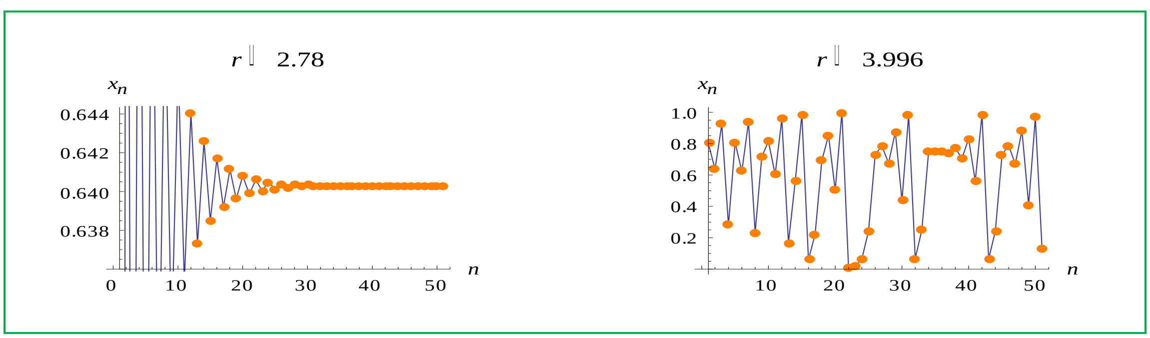

Fig. 15: Chaotic behavior of the map (1) for iteration number, $n = 50$ and $r = 2.78, 3.996$.

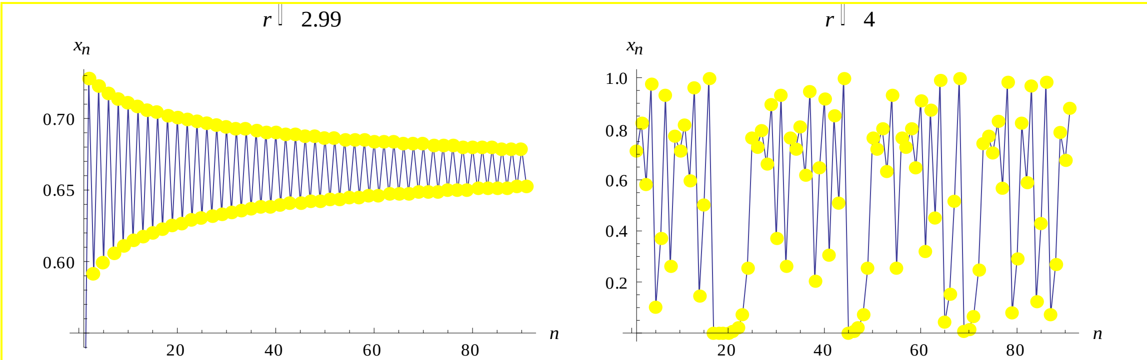

Fig. 16: Chaotic behavior of the map (1) for iteration number, $n=90$ and $r=2.99,\ 4.$

Small windows of periodic behavior appear when particular ranges of chaotic behavior rise. In a deterministic system, chaos is the sensitive initial condition dependence of periodic long-term behavior. It is said that long-term trajectories that never arrive at stable sites or periodic orbits are uncommon. Rotation is the obvious answer to an equation; everything is precise and definite. Sensitive to beginning conditions with time, points on trajectories that are near to one another gradually diverge. Around $\frac{r-1}{r}$, the system is stable when $1<r<3$ is considered. At well-defined values of r, oscillations begin to occur when r is greater than three, and their period tends to double with each successive increase in r. As the number reaches 3.57, the doubling rate increases and eventually stops being periodic. For some ranges of r, chaotic activity with brief intervals of periodic behavior emerges as the number climbs.

3.4 Dynamics of logistic iterative map for different control parameter values (Eric, 2008).

The behavior of this map is not always obvious. John von Neumann first proposed a random number generator based on the logistic map (1) in the late 1940s, but it wasn't until the subsequent research by W. Ricker in 1954 and the subsequent in-depth analytical exploration of logistic maps incipience in the 1950s with Paul Stein and Stanislaw Ulam that its complex properties beyond simple ambivalent behavior became apparent function x^4. John von Neumann proposed applying the logistic map (1) to generate numbers at random in the late 1940s. Logistic function x^0 and its iterated counterparts x^1, x^2, x^3, and x^4 as well as for varying values of the control parameter; for example, the value of the iterate four iterations later can be obtained by inputting the beginning value on the horizontal function x^4. The Pomeau-Manneville scenario describes the evolution of the unpredictable behavior of the logistic sequence as the parameter r fluctuates from about 3.57 to about 3.833, and it is characterized by a periodic (laminar) phase interrupted by bursts of a periodic activity. Other ranges oscillate between 5 esteems, etc.; all oscillation periods befall for a particular r parameter.

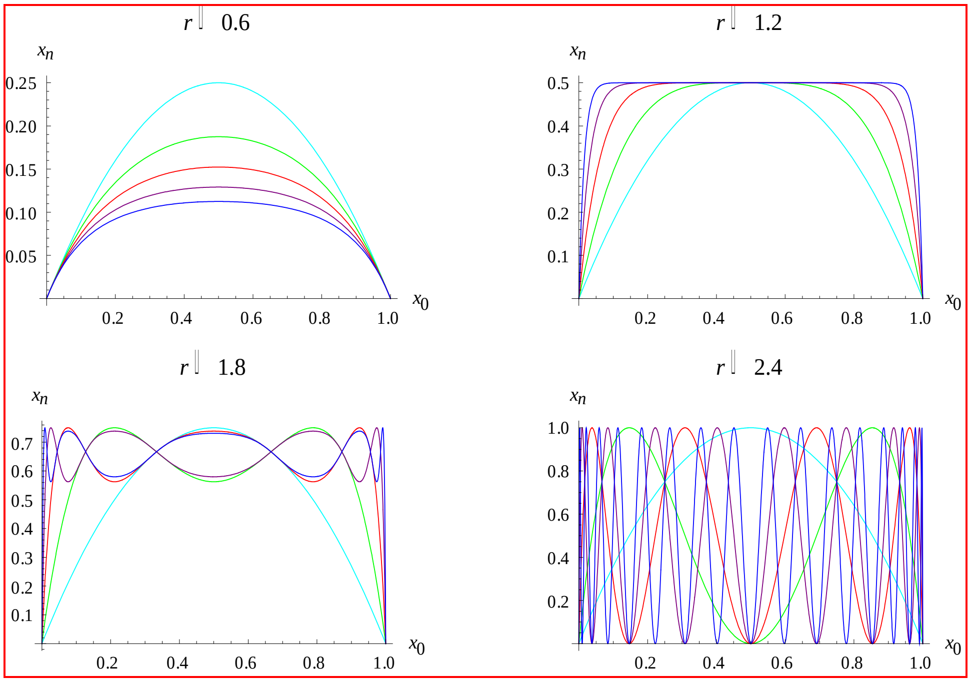

Fig. 17: Dynamics of the iteration map x^0, x^1, x^2, x^3 & x^4 for $r = 0.6, 1.2, 1.8 \& 2.4$.

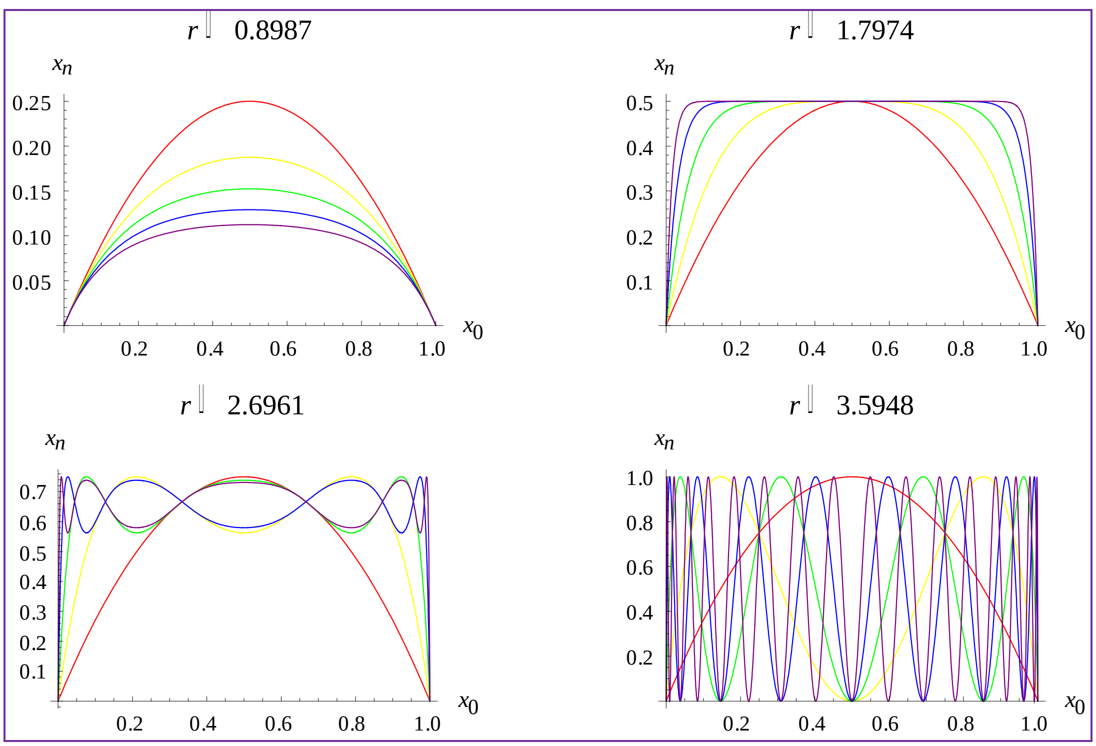

Fig. 18: Dynamics of the iteration map x^0, x^1, x^2, x^3 & x^4 for r = 0.8987, 1.7974, 2.6961 & 3/5948

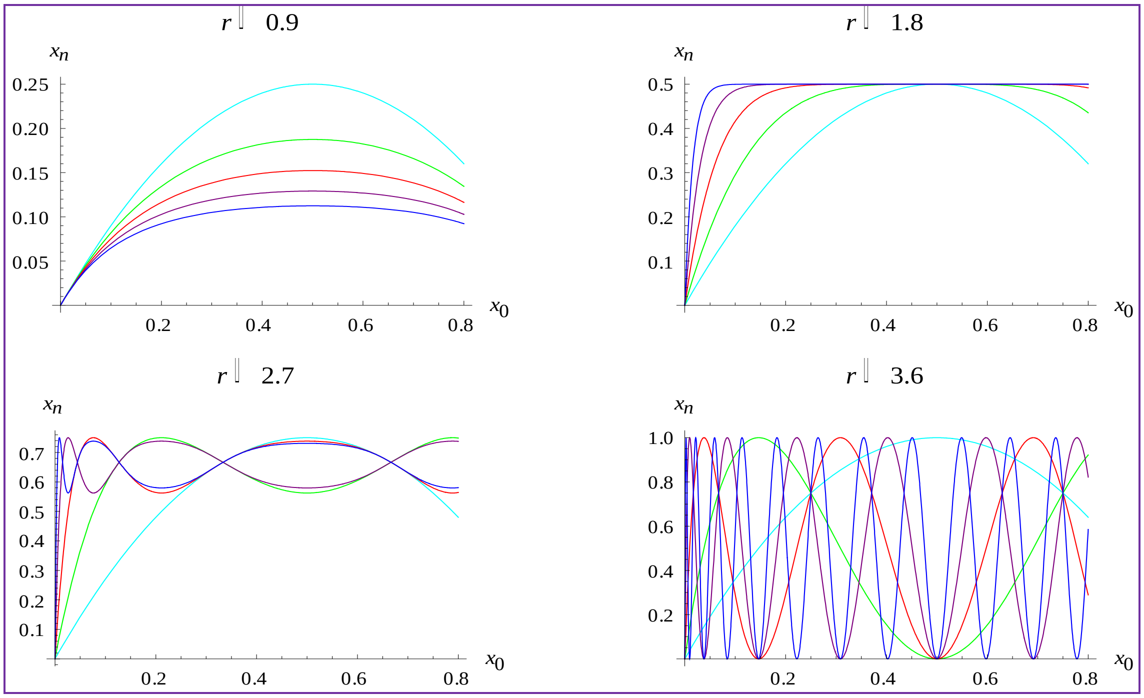

Fig. 19: Dynamics of the iteration map x^0, x^1, x^2, x^3 & x^4 for $r = 0.9, 1.8, 2.7 \& 3.6$.

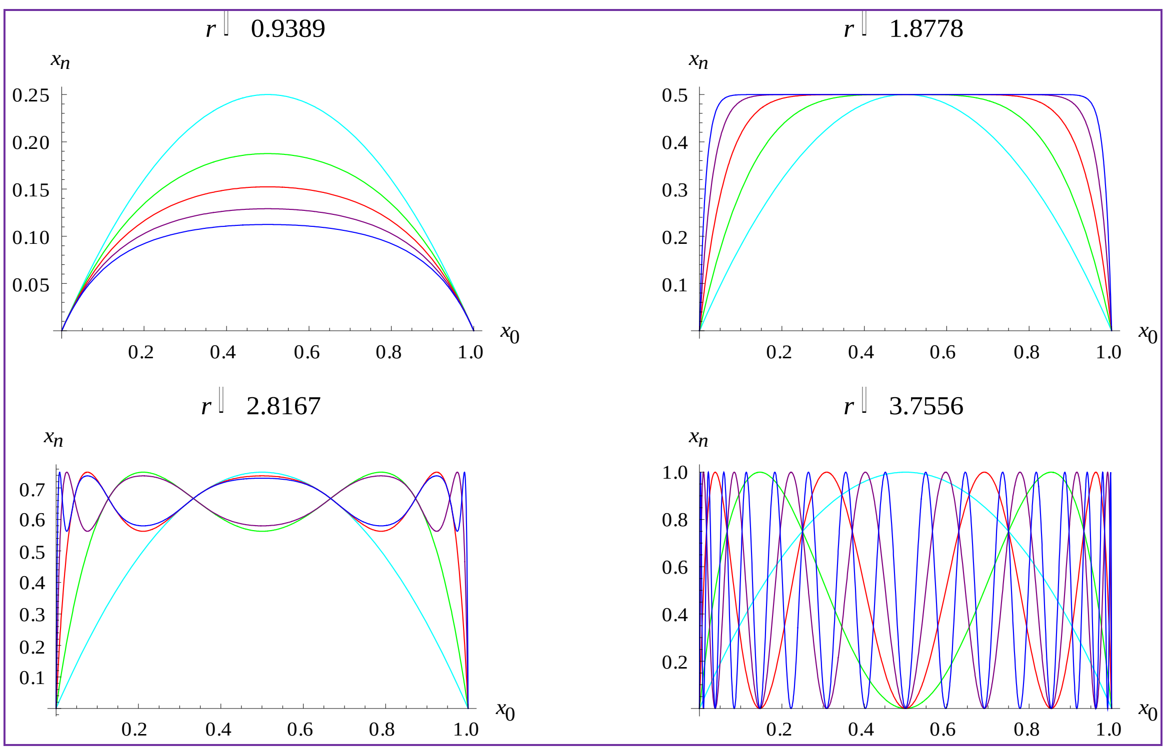

Fig. 20: Dynamics of the iteration map x^0, x^1, x^2, x^3 & x^4 for $r = 0.9389, 1.8778, 2.8167 \& 3.7556$.

Above, for a range of r values, the first value is shown through five iterations, with the number of iterations indicated by color. When it is between 1 and 2, the population soon converges on the value $\frac{r-1}{r}$ regardless of the beginning population size. A population with r between 2 and 3 will first experience a period of fluctuation about that number before arriving at the true value of $\frac{r-1}{r}$. For all values of r except 3, the rate of convergence is linear. For this parameter, convergence is substantially more sluggish than linear. The population tends toward steady oscillations between two values when r is between 3 and 3.7556. In both cases, r has a role in the outcome.

3.5 The modification of the continuous equation to a discrete logistic map (Eric, 2008).

To model population growth, Pierre Verhulst first proposed the logistic equation. It is also occasionally referred to as the Verhulst model or the logistic growth curve (1845, 1847). The model is continuous in time; however, a modification of the continuous equation to a discrete quadratic recurrence equation known as the logistic map is also extensively employed. The logistic map can be found in a lot of different mathematical contexts. The differential equation characterizes the continuous form of the logistic model.

$$\frac{dN}{dt} = rN\left(1-\frac{N}{K}\right)$$

(2)

In this equation (2), r represents the maximum rate of population expansion, while K represents the carrying capacity (i.e., the maximum sustainable population). When the two sides are divided by K and x is defined as $\frac{N}{K}$, the differential equation (3), which is also known as the logistic equation, has a solution. This solution

can be found when the logistic equation is used and is

$$x(t) = \frac{1}{1+\left(\frac{1}{x_0}-1\right)e^{-rt}} \quad (3)$$

The name "sigmoid" is sometimes applied to the function (3) and a logistic map is a discrete representation of the logistic equation (3).

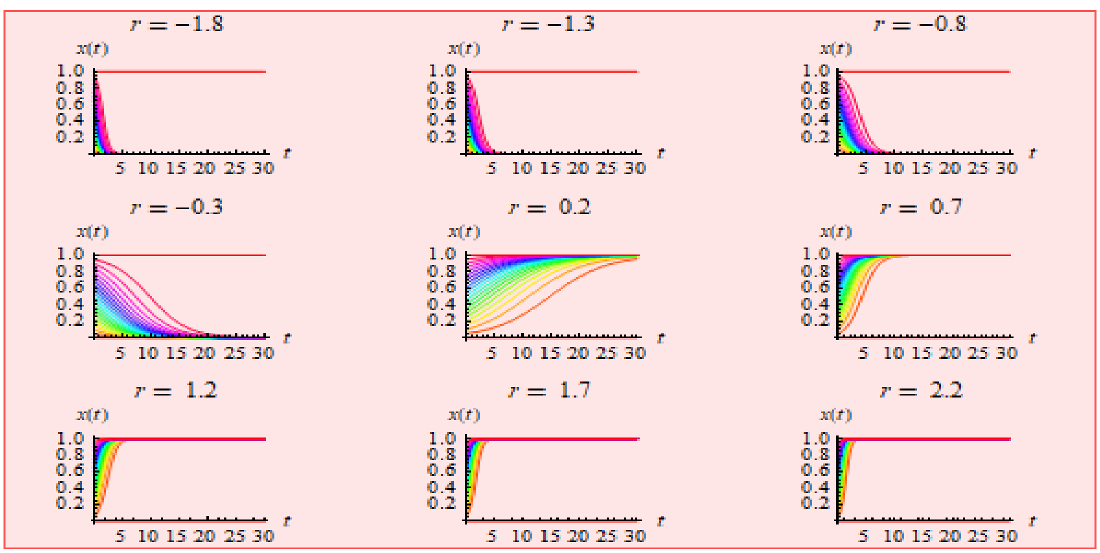

Fig. 21: Dynamics of the equation (3), for $r = -1.8, -1.3, -0.8, -0.3, 0.2, 0.7, 1.2, 1.7 \& 2.2$.

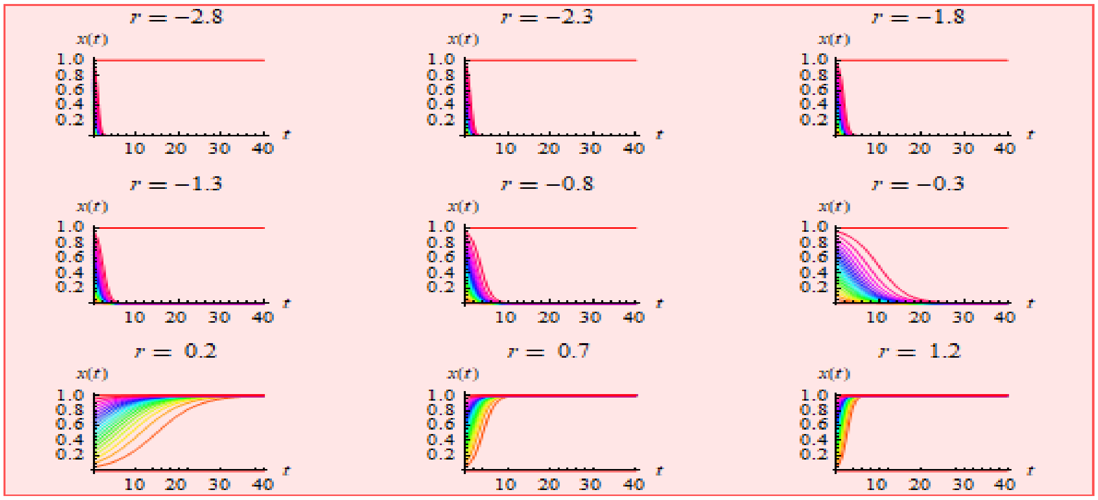

Fig. 22: Dynamics of the equation (3), for $r = -2.8, -2.3, -1.8, -1.3, -0.8, -0.3, 0.2, 0.7 \& 1.2$.

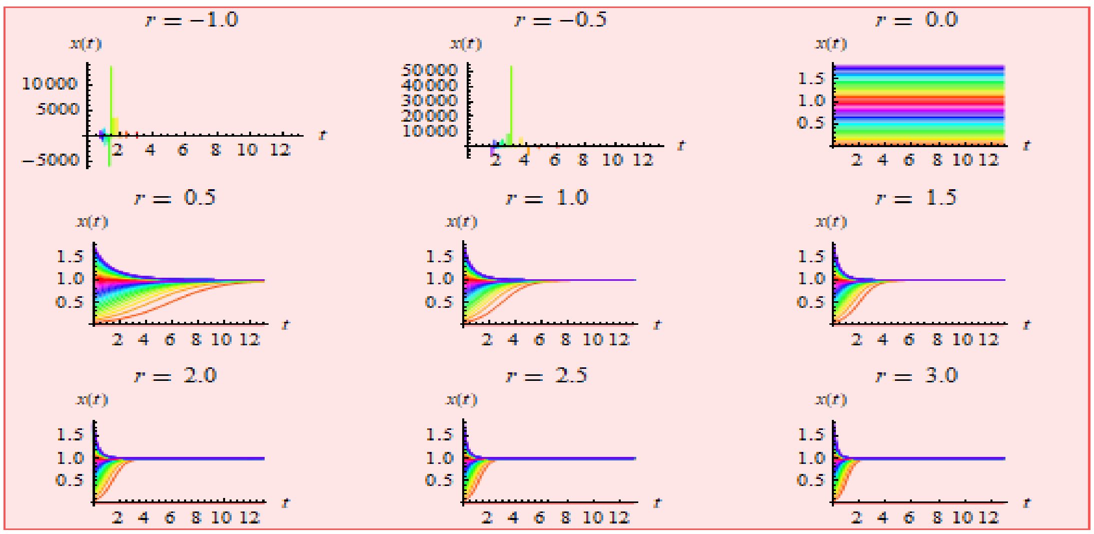

Fig. 23: Dynamics of the equation (3), for $r = -1.0, -0.5, 0.0, 0.5, 1.0, 1.5, 2, 2.5 \& 3.0$.

Plots of the previously mentioned solution are displayed for a variety of positive and negative values of beginning conditions ranging from 0.00 to 1.5 in steps 0.08 in figure 21, the initial conditions ranging from 0.00 to 2 in steps 0.09 in figure 22, and the initial conditions ranging from 0.00 to 1.8 in steps of 0.07, in figure 23 although it is needed that r is in the positive most of the time, there are circumstances in which this is not the case. Due to the fact that r is greater than zero, the only possible nonnegative equilibrium is $x = 0$ for values of r less than one. The state of balance is steady at the moment. Since $x = 0$ becomes unstable in figure 23. When $1 < r < 3$, the steady-state value of $x = \frac{r-1}{r} > 0$ is unchanging. It is observed that the $x = 0$ state is unstable while the $x = 1$ state is stable. As can be seen in the pictures above, $x(t)$ expands exponentially with time for any small beginning value of x. The equation (3) handles the chaotic behavior that occurs when the initial value of x is increased as unpredictable.

3.6 Graphical depictions of the different locations of bifurcation that can be found on the logistic map (Eric, 2008).

All bifurcation diagrams include distinct darker curves. Investigating how the dynamical characteristics of a single controlled parameterized logistic Map (1) above are derived from charting the 300th through 600th iterations of 0.5, as shown in the examples below. The parameter value r is shown along the horizontal axis, and the x-value is plotted along the vertical axis

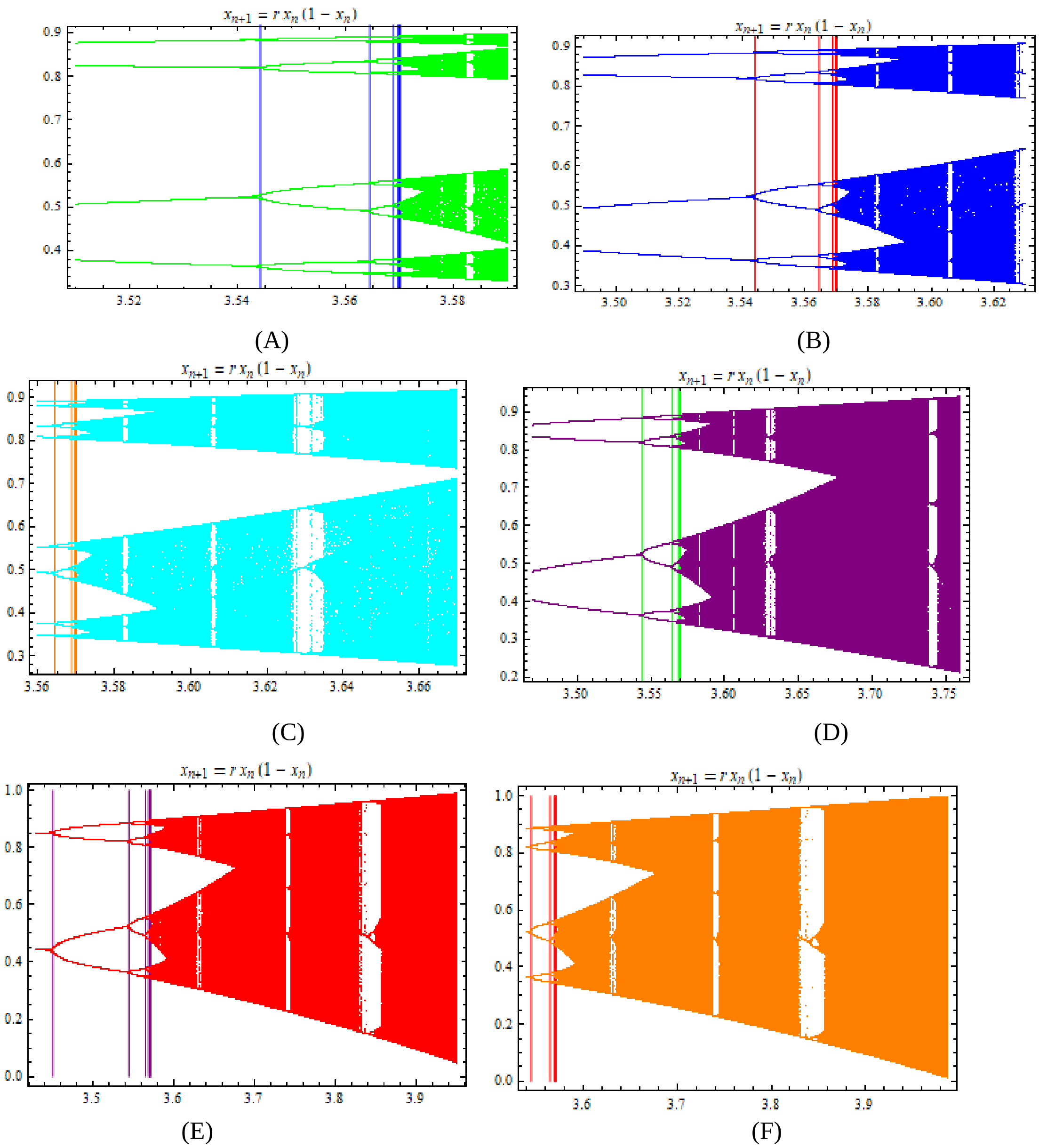

Fig. 24: Bifurcation diagrams of the equation (1): (A) $3.51 \le r \le 3.59$, (B) $3.49 \le r \le 3.63$, (C) $3.56 \le r \le 3.67$, (D) $3.47 \le r \le 3.76$, (E) $3.43 \le r \le 3.951$ & (F) $3.54 \le r \le 3.988$.

A range of 2–4 is permitted for the parameter value. Over time, x values are seen to fluctuate between 0 and 1. There is a bifurcation diagram for x_n within the range of $3.43 \le r \le 3.98$ up there. Pay attention to the white bars that go vertically. Attracting periodic points are represented by these strips. Parameter values where the attractive points for periods 5 and 6 are located are two of the most glaring examples. The above diagram is based on the region of the diagram containing a cluster of five periodic points. The expanded view shown in the bottom diagram emphasizes this zone. Take note of the region's resemblance to the bifurcation seen in figure 24. Bifurcations are period doublings, quadrupling, etc., that controls chaos in dynamical systems. It's when a nonlinear system's solution suddenly changes as a parameter is changed. As r is altered, bifurcations occur at the blue lines from the figure (A), red lines from the figure (B), orange lines from the figure (C), green lines from the figure (D), purple lines from the figure (E), and, orange lines from the figure (F) of the logistic map.

4. Conclusion

The time it takes to travel from one location to another that is on a different path increases. Trajectories that never come to a complete stop and orbits that repeat themselves fall consistently into this category.

- Since 3.56 are not periodic, the doubling time speeds up. Chaos dominates as it expands, but periodic behavior is visible in some areas.
- Our quick investigation will show if the logistic map has two or more stable nodes. If the logistic map has more than two stable nodes, our brief analysis will reveal it.
- If the initial value of r is greater than one but less than three consecutive points, then the flow will stabilize at some x greater than zero. The fixed point, however, will diverge into a limit cycle of period 2 for r greater than 3.
- At even larger values of r, there is a second fork that results in a limit cycle with four periods.
- As r increases, the values of r continue to approach one another, and the time required to reach these increasingly close values of r continues to increase by a factor of two.
- When the period reaches infinity, around the value of $r = 4$, it begins to behave erratically. This process keeps going on forever.
- In the context of a logistic map, the initial value is of the utmost importance.
- It is clear that the change and chaotic behavior of the logistic map is not only dependent on the parameters, but also on the number of iterations that are performed.
- In the range of 3.43 and 4, this map can be used to detect chaotic situations for different iteration numbers.

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