JnUJSci

Jagannath University Journal of Science

Volume 09, Number II, December 2022, pp. 76-84

https://jnu.ac.bd/journal/portal/archies/science.jsp

ISSN 2224-1698

A New Maxmin Approach to Solve Time Minimization Transportation Problem with Mixed Constraints

Research Article

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Received: 20 February 2023, Accepted: 29 May 2023

ABSTRACT

Some emergency cases transportation of goods is essential than transportation cost. Regarding the issue this paper presents an efficient algorithm to take full advantage of the transported unit with minimum time pass over the shipping cost in which the problem consists of equality and inequality constraints. Furthermore, numerical examples are used to demonstrate the method. A numerical example demonstrates how our suggested algorithm is very simple to comprehend and gets the most shipping products per unit.

Keywords: *Transportation problem, Time minimization Transportation problem, Mixed constraints, Maximum flow, Minimum time*

1. Introduction

Transportation problem (TP) is a particular type of linear programming issue, and closely related to moving goods from one place to another. The goal is to determine the minimal amount of good that has to be delivered from each source to a particular destination while also meeting all supply and demand constraints for the relevant sources and destinations. It is argued that there are equality restrictions on a balanced transportation problem if the overall supply and demand values are equal.

French mathematician Monge (1781) formalized the transportation issue. Russian mathematician and economist L. Kantorovich (1958) made significant advancements in the discipline during World War II. Hitchcock (1941) was the one who originally presented the issue in its classic form. From the first introduction, different efficient methods of solution illustrated by Koopman (1947), Dantzig (1951) and then Charnes et al. (1953).

The total transportation problem is converted to Transportation Problem with Mixed Constraints (TPMC) if the supply of a source is significantly extended or lowered and the demand of a destination is similarly substantially increased or diminished. For More clarification, In the case of a general transportation issue, a factory could be required to produce 20 units of a product, but owing to an unforeseen circumstance, it might be able to produce fewer or more than 20 units instead. Similar to this, when a warehouse needed 40 pieces

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of inventory, they unexpectedly needed more or fewer units. That is why there is a transportation issue with several limits. Because of this, the transportation issue differs little from the balanced transportation issue. As a result, in real life, the majority of transportation issues include many limitations.

Brigden (1974) first carefully examined a unique sort of TPMC. Numerous research work on TPMC are found in the literature of Klingman et al. (1974), Isermann (1982), Adlakha et al. (2000, 2006), Pandian et al. (2010a-2010d), Mondal et al. (2012, 2014, 2015), Akilbasha et al.(2017).

The primary objective shouldn't be to cut transportation costs. In reality, a crisis situation might arise when the cost of expanding a moved unit is comparable. We would want to describe this urgent situation: itis very essential to send disaster relief to an area that is affacted by a sudden hurricane, earthquak, tsunami or flood. Some examples are mountain collaps, building collapse like Rana Plaza collaps or a daring rescue mission, suddenly happened earthquake in Turkey etc. When a medical emergency arises (such as corona, diarrhea, typhoid, dengue, chikungunya, or any other disease in epidemic form), enough medication or food should be sent to the affected region as quickly as feasible.

The best disaster assistance may readily compare to consider cost in this emergency situation. We provide a way to transfer a maximum number of units while meeting all supply and demand constraints in the shortest amount of time feasible without considering the cost of transportation.So, Time Minimization Transportation Problem arise.

Time Minimization Transportation Problem (TMTP) with equality constraints addressed by Peter L. Hammer for the first time in 1969. Then day by day many research worked found such as Garfinkel and Rao (1971), Wlodzimierz Szwarc (1971).A good numbers of paper has already been published on TMTP with equality requirements such as:Bhatia (1977), Sharma, J.K. and Swarup, K., (1977), Pandian and Natarajan (2011) , Uddin, M.S., (2012), Nikolić, I., (2007),Md. Main Uddin et al. (2013), Mollah Mesbahuddin Ahmed (2014, 2015), Khan, A. R. et. al. (2016), Khan, A.R. (2017) have shown TMTPs and its variants in their research. They presenteddifferent algorithms for solving TMTP with equality constraints.

From this survey of the literature, we can see that while there has been a sizable amount of study into finding the best solution to the time-minimizing transportation problem considering equality constraints, relatively few studies have focused on the issue of mixed constraints.The Time Minimization Transportation Problem with Mixed Constraints (TMTP-MC) is extensively useful in many real life situations where distinct sources and destinations require variableamounts of supply and demand commoditiesrespectively depending on to their availability and requirements.Agarwal, S., & Sharma, S. (2014) as far I know (probably) was first introduced TMTP-MC. Agarwal, S., & Sharma, S. (2018, 2020) finding IBFS of TMTP-MC. Rashid, F. et al. (2021) proposed a method to solve TMTP-MC. Mentioned above literature review on TMTP-MC we see that all methods are focused on to find minimum time. But in this paper we focused our method to maximize transported unit along with minimum time.

2. Formulation of General Transportation Problem

We represent the arrangement of m source points from which a good is transported by $O_1, O_2, \cdots\cdots, O_m$ and the arrangement of n destination points by $D_1, D_2, \cdots\cdots, D_n$ to which the product is transported. Supply point i can supply at most s_i units $(i = 1, 2, \cdots\cdots, m)$ whereas demand point j must receive at least d_j units $(j = 1, 2, \cdots\cdots, n)$ of shipped good. x_{ij} is the amount transported and c_{ij} is the unit transportation cost between supply point i to destination point j .Bearing in mind the overhead symbolization, the general TP can be characterized by the organization as in the following Fig.1:

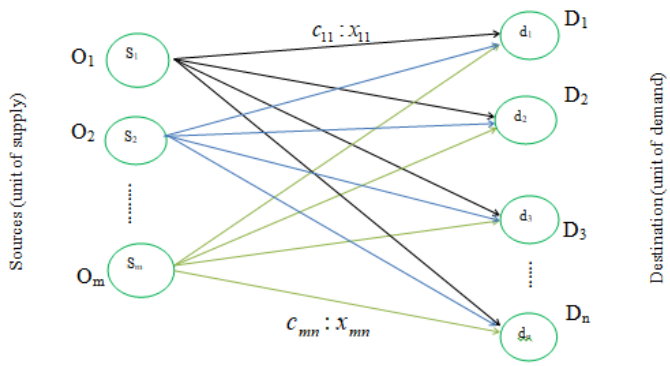

Fig. 1: Network Representation of Transportation Problem

Also depiction of TP by using matrix form as follows:

Table1: Matrix Representation of TP.

Factory	Warehouse				Supply
	1	2	…	n	
1	x_{11} c_{11}	x_{12} c_{12}	…	x_{1n} c_{1n}	s_1
2	x_{21} c_{21}	x_{22} c_{22}	…	x_{2n} c_{2n}	s_2
⋮	⋮	⋮	…	⋮	⋮
m	x_{m1} c_{m1}	x_{m2} c_{m2}	…	x_{mn} c_{mn}	s_m
Demand	d_1	d_2	…	d_n	

The TP is then expressed mathematically as follows:

Minimize $z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$.

Subject to $\sum_{j=1}^{n} x_{ij} \le s_i$;

$i = 1, 2, \cdots\cdots, m$ (Supply constraints)

$\sum_{i=1}^{m} x_{ij} \ge d_j$; $j = 1, 2, \cdots\cdots, n$ (Demand constraints)

$x_{ij} \ge 0$. for all $i = 1, 2, \cdots\cdots, m$ and $j = 1, 2, \cdots\cdots, n$.

For Balanced TP (1) becomes to:

Minimize $z = \sum_{i=1}^{m} \sum_{j=1}^{n} c_{ij} x_{ij}$.

Subject to $\sum_{j=1}^{n} x_{ij} = s_i$;

$i = 1, 2, \cdots\cdots, m$ (Supply constraints)

$\sum_{i=1}^{m} x_{ij} = d_j$; $j = 1, 2, \cdots\cdots, n$ (Demand constraints)

$x_{ij} \ge 0$; for all $i = 1, 2, \cdots\cdots, m$ and $j = 1, 2, \cdots\cdots, n$.

2.1. Formulation of TP with mixed constraints

Mathematically, a TPMC is defined as follows:

$$Minimize\ Z = \sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij}$$

Subject to Supply Constraints

$$\sum_{j\in T} x_{ij} = a_i, \quad i \in S_1$$

$$\sum_{j\in T} x_{ij} \geq a_i, \quad i \in S_2$$

$$\sum_{j\in T} x_{ij} \leq a_i, \quad i \in S_3$$

And $x_{ij} \geq 0$

Where S_1, S_2, S_3 are partitioned from S, $i \in S$, $i = 1, 2, \cdots\cdots, m$

And

Demand Constraints

$$\sum_{i\in S} x_{ij} = b_j, \quad j \in T_1$$

$$\sum_{i\in S} x_{ij} \geq b_j, \quad j \in T_2$$

$$\sum_{i\in S} x_{ij} \leq b_j, \quad j \in T_3$$

Where T_1, T_2, T_3 are partitioned from T, $j \in T$, $j = 1, 2, \cdots\cdots, n$

This is the standard form of TPMC.

2.2. Formulation of Time Minimization TP with Mixed constraints (TMTP-MC):

The structure of TMTP-MC is similar to the regular cost minimization problem with mixed constraints, except that the unit transportation cost c_{ij} is replaced by the unit transportation time t_{ij} required to effect a complete shipment of the goods from specific source *i* to particular destination *j*.

Table : 2 *Matrix Representation of TMTP-MC*

Factory	Warehouse				Supply
	1	2	$\cdots$	n	
1	x_{11} t_{11}	x_{12} t_{12}	$\cdots$	x_{1n} t_{1n}	$\geq/=/\leq a_1$
2	x_{21} t_{21}	x_{22} t_{22}	$\cdots$	x_{2n} t_{2n}	$\geq/=/\leq a_2$
$\vdots$	$\vdots$	$\vdots$	$\cdots$	$\vdots$	$\vdots$
m	x_{m1} t_{m1}	x_{m2} t_{m2}	$\cdots$	x_{mn} t_{mn}	$\geq/=/\leq a_m$
Demand	$\geq/=/\leq b_1$	$\geq/=/\leq b_2$	$\cdots$	$\geq/=/\leq b_n$	

x_{ij} is the quantity shipped from source *i* to demand point *j* and t_{ij} per unit transportation time from sources *i* to demand point *j*.

$i = 1, 2, \cdots\cdots, m$ and $j = 1, 2, \cdots\cdots, n$

Therefore the LPP model of TMTP-MC is

$Minimize\ Z$: Max $t_{ij} | x_{ij} > 0$

Subject to Supply Constraints

$$\sum_{j \in T} x_{ij} = a_i, \quad i \in S_1$$

$$\sum_{j \in T} x_{ij} \geq a_i, \quad i \in S_2$$

$$\sum_{j \in T} x_{ij} \leq a_i, \quad i \in S_3$$

And $x_{ij} \geq 0$

Where S_1, S_2, S_3 are partitioned from S, $i \in S$, $i = 1, 2, \cdots\cdots, m$

And

Demand Constraints

$$\sum_{i \in S} x_{ij} = b_j, \quad j \in T_1$$

$$\sum_{i \in S} x_{ij} \geq b_j, \quad j \in T_2$$

$$\sum_{i \in S} x_{ij} \leq b_j, \quad j \in T_3$$

Where T_1, T_2, T_3 are partitioned from T, $j \in T$, $j = 1, 2, \cdots\cdots, n$

This is the standard form of TMTP-MC.

Basic Assumptions:

1. S_2 must allocate at least a_i units of supply which is equal to the total minimum demand and T_2 must receive at least b_j units which is equal to the total minimum supply.
2. They start concurrently from their respective origins.
3. The time t_{ij} is independent of the quantity x_{ij} .

3. Theoretical Development of Proposed Method.

Step-1: Formulation of TMTP-MC.

Step-2: For identifying total minimum supply and total minimum demand we set the condition below:

$a'_i = a_i$, $\boldsymbol{i} \in \boldsymbol{S_1} \cup \boldsymbol{S_2}$

$a'_i = 0$, $\boldsymbol{i} \in \boldsymbol{S_3}$, $i = 1, 2, \cdots\cdots, m$, $j = 1, 2, \cdots\cdots, n$

and

$b'_i = b_i$, $\boldsymbol{i} \in \boldsymbol{S_1} \cup \boldsymbol{S_2}$

$b'_i = 0$, $\boldsymbol{i} \in \boldsymbol{S_3}$, $i = 1, 2, \cdots\cdots, m$, $j = 1, 2, \cdots\cdots, n$

and form a new TMTP-MC.

Step-3:

Calculate total minimum supS_{min}andtotal minimum demand D_{min}

$S_{min} = \sum a_i$, $i = 1, 2, \cdots\cdots, m$

$D_{min} = \sum b_j$, $j = 1, 2, \cdots\cdots, n$

Step-4: According to our assumption the supply and demand constraints will be

Supply Constraints

$$\sum_{j \in T} x_{ij} = a_i, \quad i \in S_1$$

$$\sum_{j \in T} x_{ij} = D_{min}, \quad i \in S_2$$

$$\sum_{j \in T} x_{ij} = a_i, \quad i \in S_3$$

and

Demand Constraints

$$\sum_{i \in S} x_{ij} = b_j, \quad j \in T_1$$

$$\sum_{i \in S} x_{ij} = S_{min}, \quad j \in T_2$$

$$\sum_{i \in S} x_{ij} = b_j, \quad j \in T_3$$

Finally, get the new supply S'_{max} and new demand D'_{max}.

$$S'_{max} = \sum a_i + \sum D_{min} \quad , i=1,2, \cdots\cdots, m$$

$$D'_{max} = \sum b_j + \sum S_{min} \quad , j=1,2, \cdots\cdots, n$$

Step-5: If necessarily make it balance adding zero row or column.

Step-6: Locate maximum time. Let us choose the row and column that pass the maxima time. From maximum time to minimum time allocate maximum possible allotment. If two or more minimum time exist choose which cell get maximum amount. Then choose next minimum time either same row or column and allocate as amount as possible and crossed out the row and column.

Step-7: If all supply and demand exhausted, then stop.

Step-8: Find total maximum transported unit $\sum x_{ij}$ and minimum required time.

4. Numerical Illustration:

Example 1

For an emergency situation such as an Earthquake, government opened five emergency relief collection booths in five corners (airport) of Turkey. They collect medicine/food/necessary accessories from three reputed donating countries such as U, S and A are situated at different distances from Turkey. Production capacity of U is exactly 80 units, A at most 140 units and that of S has at least 120 units. Moreover, collection booth-1 having a limit of demand exact 11-unit, booth-2 having a limit of demand at least 13 unit, booth-3 having a limit of demand exact 60 units, booth-4 having a limit of demand at most 80 units and collection booth-5 having a limit of demand at least 80 units. Unit transportation time from source to collection booth are given below by this matrix.

From this type of TP with mixed constraints and we conclude the maximun transported unit with minimum transportation time:

Step-1: Formulation of Time Minimization Transportation Problem with Mixed Constraints (TMTP-MC).

		Collection Booth					Supply
		D_1	D_2	D_3	D_4	D_5	
Country	U	8	6	8	6	10	=80
	S	9	8	9	6	7	≥120
	A	11	10	8	13	9	≤140
Demand		=40	≥40	=60	≤80	≥80	116

Step-2, Step-3 and Step-4:

		Collection Booth					Supply	New min supply S_{min}	New max supply S'_{max}
		D_1	D_2	D_3	D_4	D_5			
Country	U	8	6	8	6	10	=80	80	80
	S	9	8	9	6	7	≥120	120	220
	A	11	10	8	13	9	≤140	0	140
Demand		=40	≥40	=60	≤80	≥80			
New min. demand D_{min}		40	40	60	0	80		200/ 220	
New max. demand D'_{max}		40	200	60	80	200			440/ 580

Step-5:

New Equivalence StandardTime Minimization Transportation Problemwith Mixed Constraints will be according to our method:

		Collection Booth					Supply
		D_1	D_2	D_3	D_4	D_5	
Country	U	8	6	8	6	10	80
	S	9	8	9	6	7	220
	A	11	10	8	13	9	140
Demand		40	200	60	80	200	440/580

It is unbalance form of TP. Make it balance we get

		Collection Booth					Supply
		D_1	D_2	D_3	D_4	D_5	
Country	U	8	6	8	6	10	80
	S	9	8	9	6	7	220
	A	11	10	8	13	9	140
		0	0	0	0	0	140
Demand		40	200	60	80	200	580

Step-6, Step-7 and Step-8:

According to our proposed rule in Step-6, Step-7, Step-8 , finally we get the table below:

	D_1	D_2	D_3	D_4	D_5	Sup.
O_1	8	80 6	8	6	10	=80
O_2	40 9	120 8	9	6	60 7	≥120
O_3	11	10	60 8	13	80 9	≤140
Dem.	=40	≥40	=60	≤80	≥80	440

Therefore, the solution for the given problem is $x_{12} = 80$, $x_{21} = 40, x_{22} = 120$, $x_{25} = 60$, $x_{33} = 60$, and $x_{35} = 80$ and the corresponding time of the cells are $t_{12} = 6$, $t_{21} = 9t_{22} = 8$, $t_{25} = 7$, $t_{33} = 8,$ and $t_{35} = 9$.

Therefore, the total transportation time required

$$\boldsymbol{T_0} = \boldsymbol{max}\{t_{12}, \quad t_{21}, \ t_{22}, \quad t_{25}, t_{33}, t_{35}\}$$
$$= max\{6, 9, \ 8, \ 7, \ 8, \ 9\} = 9$$

So, the total transported unit is 440 and required maximum time 9.

5. Conclusions

When transporting urgent materials, the amount transported matters more than the transportation cost. We developed an algorithm that will shipped maximum unit in minimum time satisfying all supply and demand requirements while over passing the transportation cost. The method proposed in this paper is very simple and easy understandable. No significant paper has been published on this context yet, on the contrary, our solution technique is efficient and justified as the used data in the paper real. It provides solution in a simple manner, and examination could be helpful for troughs in pursuing vital choices to managing the emergency situation.

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